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ABSTRACT

The purpose of this project is to help teachers of mathematics use the computer to aid the teaching and learning of many mathematical concepts in a meaningful way. To this end, commercially available software packages are described which are versatile, cost effective, and easy to use. A review and several lessons, each including homework exercises, have been written for each package and are presented in ready-to-use form. Chapter 1, "Geometric Supposer: Triangles" (Sunburst), includes five lessons, such as types of triangles, angle sums, midsegments, similar triangles, and perpendiculars from a random point. Chapter 2, "Graphing Equations" (Conduit), consists of four lessons of linear equations including an equation and its graph, functions from a given slope, y-intercept or points, and reviews. Chapter 3, "Discovery Learning in Trigonometry" (Conduit) is a lesson about polar equations. Chapter 4, "Interactive Experiments in Calculus" (Prentice-Hall), contains three lessons about the nature of polynomial functions and techniques of numerical integration. (YP)

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PROJECT TO INCREASE MASTERY OF MATHEMATICS AND SCIENCE

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A REVIEW

OF

SELECTED MICROCOMPUTER SOFTWARE PACKAGES

WITH LESSONS

FOR TEACHING MATHEMATICS

GRADES 8 - 12

[Volume 1]

A Curriculum Development Project
of the
Project to Increase Mastery of Mathematics and Science

APRIL 1987

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A REVIEW

OF

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WITH LESSONS

FOR TEACHING MATHEMATICS

GRADES 8 - 12

A Curriculum Development Project

of the

Project to Increase Mastery of Mathematics and Science

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APRIL 1987

HISTORY, GOALS, AND SUPPORT OF PIMMS

The Project to Increase Mastery of Mathematics (PIMM) began in 1979 as a joint effort of a diverse group of volunteers--individuals from education, business, government, industry, and labor--to foster greater command, understanding, and appreciation of mathematics by graduates of Connecticut schools. The symbiotic relationship of mathematics and its applications was a pervasive theme of the various PIMM undertakings.

In 1983, the organization broadened its scope to become the Project to Increase Mastery of Mathematics and Science (PIMMS), and at the same time expanded its activities by mounting a Fellowship Program: notably able Connecticut high school teachers of mathematics and science were invited to study together at Wesleyan in courses and seminars as a prelude to offering consultative assistance on curricular matters to schools requesting such service and to providing workshops to teachers for their professional development.

Several groups of Fellows are devoting the bulk of their energies to writing projects. The materials they prepare will be broadly disseminated to facilitate the introduction into school curricula of mathematical applications of major importance and considerable attractiveness.

Now several Fellowship Programs are in operation, extending the concept to middle school and elementary school teachers. A significant value of the Fellowship Programs is found in the colleagueship developed among professionals working in different institutions at varied levels.

Throughout its growth, PIMMS has continued as a partnership of individuals and organizations. The list of devoted volunteers is too long to be recorded here. Major financial support of PIMMS has been supplied by the Connecticut Department of Education, the Connecticut Department of Higher Education, the Charles E. Culpeper Foundation, the General Electric Foundation, GTE, Divisions of United Technologies Corporation (Hamilton Standard, Norden Systems, Otis Elevator, Pratt and Whitney Manufacturing, Research Center, Sikorsky Aircraft), and Wesleyan University. Substantial contributions have also been made by Amax Foundation, Army Research Office, Boehringer Ingelheim, Chesebrough-Ponds, Choate Rosemary Hall, CIGNA, Connecticut Academy of Science and Engineering, General Signal, the Edward W. Hazen Foundation, Northeast Utilities, Omega Group, Pfizer Inc., Raymond Engineering, SNET, Stanley Works and Union Carbide Corporation.

All of us who are concerned for high quality education owe a great debt of gratitude to these organizations for their corporate citizenship.

R.A. Rosenbaum
Director, PIMMS

PIMMS curriculum development publications currently available are:

* Apple Microcomputer Software Evaluation, August, 1985
* A Guide to Curriculum Development in Applied Mathematics: A Three-year Sequence for Noncollege Bound Students, 82 pp., Fall 1986
* Down to Earth Chemistry, (for the Noncollege Bound), 3 sample units, 60 pp., Feb. 1987
* A Review of Selected Microcomputer Software Packages, with Lessons, 122 pp., April 1987
* Newsletter #3, 10 pp., April 1987

FOREWORD

In the report, How Schools Use Microcomputers, Henry Jay Becker stated, "... with an organized and sustained strategy for using computers to improve the effectiveness of school instruction we might accomplish what has eluded education throughout recent decades -- clear and obvious improvements in student accomplishment and in how much learning occurs in a day in school. Nothing would help the institution or its practitioners more."*

The preparation of this volume, and subsequent editions to follow, is a step towards the realization of that goal.

Connecticut school districts may contact the PIMMS office for information on scheduling an Awareness Workshop or other in-service program related to the use of computers for mathematics instruction.

We earnestly solicit the comments of users of this material. Please send your suggestions for improvements, reviews, or lessons to

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*Becker, Henry Jay, How Schools Use Microcomputers: Summary of the 1983 National Survey, The Johns Hopkins University Center of Social Organization of Schools; Baltimore, 1985.

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INTRODUCTION

In the late '70s and early '80s the microcomputer burst upon the scene of public education. School districts purchased the hardware and used the equipment mainly to teach various programming languages. While the potential power of this new technology was recognized, very few school systems developed programs to incorporate the use of the computer as an instructional tool.

The purpose of this project is to help teachers of mathematics get started using the computer to aid the teaching and learning of many mathematical concepts **in a meaningful way**. To this end, the writing team has chosen commercially available software packages which are versatile, cost effective, and easy to use. A review and several lessons, each including homework exercises, have been written for each package and are presented in ready-to-use form. The material may be photo-copied for classroom use.

The lessons presented in this volume are designed to promote interaction between the teacher and the students and among the students. Many lessons are developmental and should be followed by a more formal treatment of the concepts involved. Contrary to popular belief, this type of activity does not increase the time needed to teach the concepts but improves efficiency and effectiveness by promoting the understanding of a problem, the ability to visualize the problem, and the ability to verbalize a statement of the solution of the problem.

We hope that this initial volume will help to spark the imagination of creative teachers who will then begin to develop their own lessons. Although all of the software reviewed herein relates to the Apple IIe microcomputer, some of the packages may be available for other computers. Users are advised to check software catalogs for such availability. As more and better instructional software becomes available for the Macintosh and IBM microcomputers, reviews and lessons for them will be included in future editions of this publication.

It should be noted that software packages that already contain excellent instructional support material -- such as the four volumes of Compter Graphing Experiments (Addison Wesley) and Mathematics: Volume 1, distributed by the Minnesota Educational Computing Consortium (MECC)-- have not been included here; however, any school district interested in developing a core library of software for mathematics instruction should include those packages with their initial purchases.

Also, users are advised to read carefully the restrictions on use of software to assure that your use does not violate copyright regulations. Many software packages are available in laboratory sets so that a sufficient number of diskettes will be available for a laboratory activity. (Users also should be aware that even when the booting of multiple machines is allowed by the copyright rules, it is an inefficient and inconvenient practice.)

TITLE: THE GEOMETRIC SUPPOSER:TRIANGLES
PUBLISHER: SUNBURST COMMUNICATIONS INC.
ADDRESS: 39 WASHINGTON AVE.
PLEASANTVILLE N. Y. 10570
PRICE: $99.95
COMPUTER: Apple IIe, IIc
PIMMS' RATING: EXCELLENT

PROGRAM IS APPROPRIATE FOR
ABILITY GROUP : LOW, MIDDLE, HIGH
COURSE : GEOMETRY
FORMAT : CLASS DEMO/INSTRUCTIONAL, DISCOVERY/ LABORATORY
CHARACTERISTICS : USER FRIENDLY, DOCUMENTATION, ON SCREEN HELP, GRAPHICS, EDIT FUNCTIONS, DISK DOES NOT NEED TO REMAIN IN DRIVE

ADDITIONAL PERIPHERALS NEEDED: NONE

DESCRIPTION: The Geometric Supposer allows the student to do any construction which is possible with a compass and a straightedge. Measurements can be made and scales can be changed to allow students to make conjectures about properties and relationships among triangles. A REPEAT function allows students to investigate their conjectures about a previous construction by repeating it on a new triangle such as a right, acute, obtuse, isosceles, equilateral or a new triangle of the user's choice.

Students should become familiar with the Supposer by experimenting with the Supposer options including Draw, Label, Erase, Measure, Scale, Repeat, and New Triangle. The documentation accompanying the Supposer illustrates the use of each menu with simple examples, permitting its use as a tutorial.

The Supposer can be effective in motivating students to think about geometry by allowing them to discover theorems on their own without being given advance information. The teacher's role is to provide a meaningful lab activity or classroom demonstration using the Supposer to permit making discoveries which then could be followed by homework exercises including formal proofs. Students also should be encouraged to make any additional conjectures which they feel may be valid.

Additional support material will be available soon from Sunburst. Other programs in the series include the Geometric Pre-Supposer and the Geometric Supposer: Quadrilaterals. The Pre-Supposer is a very versatile program which can handle complex drawings including circles.

LESSON 1

OBJECTIVE: To develop definitions of right, acute, obtuse, isosceles, and equilateral triangles.

COURSE TITLE: Geometry

SOFTWARE NEEDED: The Geometric Supposer (Sunburst)

SUGGESTED TIME: 1 Lab Period

PREREQUISITE SKILLS: The student should be able to use the Supposer to construct different types of triangles and be able to measure the parts of a triangle.

ACTIVITY: Discovery Laboratory

TRIANGLE TYPES

LAB ACTIVITY:

Use the Supposer to draw the indicated types of triangles and make use of the repeat and/or scale function to make several constructions of each type of triangle. Fill in the missing data below and write a definition for each type of triangle.

RIGHT TRIANGLE	Triangle 1	Triangle 2	Triangle 3
m∠ABC			
m∠BAC			
m∠ACB			
AB			
AC			
BC			

OBTUSE TRIANGLE	Triangle 1	Triangle 2	Triangle 3
m∠ABC			
m∠BAC			
m∠ACB			
AB			
AC			
BC			

ISOSCELES TRIANGLE	Triangle 1	Triangle 2	Triangle 3
m∠ABC			
m∠BAC			
m∠ACB			
AB			
AC			
BC			

ACUTE TRIANGLE	Triangle 1	Triangle 2	Triangle 3
m∠ABC			
m∠BAC			
m∠ACB			
AB			
AC			
BC			

EQUILATERAL TRIANGLE	TRIANGLE 1	TRIANGLE 2	TRIANGLE 3
m∠ABC			
m∠BAC			
m∠ACB			
AB			
AC			
BC			

Use the data which was generated with the Supposer to write a definition for each of the following types of triangles.

RIGHT TRIANGLE-

OBTUSE TRIANGLE-

ISOSCELES TRIANGLE-

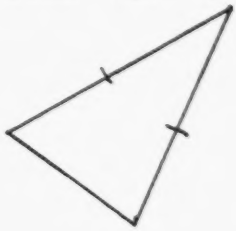
ACUTE TRIANGLE-

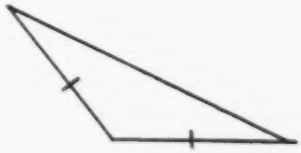
EQUILATERAL TRIANGLE-

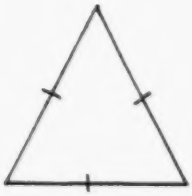
HOMEWORK EXERCISES

Classify each of the following triangles as right (r), acute (a), obtuse (o), isosceles (i), equilateral (e). Place the appropriate letter next to the figure.

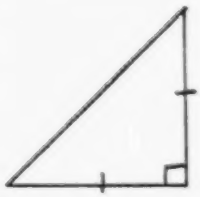
1.

2. 

3. 

4. 

5.

6. 

LESSON 1

ANSWERS TO ACTIVITIES/EXERCISES

Use the data which was generated with the Supposer to write a definition for each of the following types of triangles.

RIGHT TRIANGLE- A triangle with a right angle.

OBTUSE TRIANGLE- A triangle with an obtuse angle.

ISOSCELES TRIANGLE- A triangle with at least two sides congruent.

ACUTE TRIANGLE- A triangle with three acute angles.

EQUILATERAL TRIANGLE- A triangle with all sides congruent.

HOMEWORK EXERCISES

1. o
2. a,i
3. o,i
4. a,i,e
5. r
6. r,i

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LESSON 2

OBJECTIVES: 1. To explore the theorem stating that the sum of the measures of the angles of a triangle is 180 degrees.
2. To explore the theorem stating that the measure of an exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.

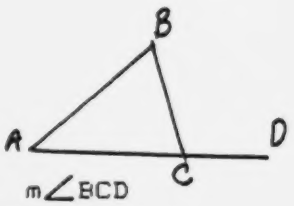
COURSE TITLE: Geometry
SOFTWARE NEEDED: The Geometric Supposer (Sunburst)
SUGGESTED TIME: 1 Lab Period
PREREQUISITE SKILLS: All students should be able to classify types of triangles and to identify the remote interior angles of a triangle with respect to a given exterior angle.

ACTIVITY: Discovery Laboratory

ANGLES OF A TRIANGLE

LAB ACTIVITY:

Use the Supposer to draw at least 2 of each type of triangle described below. Use the extension function to extend a side through a given point as shown. You may use the REPEAT function to reconstruct each type of triangle. A calculator may be used to find the sum of the angles.



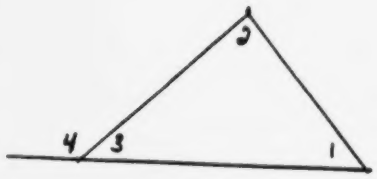
Draw	ABC	m∠ABC	m∠BAC	m∠CBA	Sum of Interior Angles	Sum of Remote Interior Angles	m∠BCD
Right							
Acute							
Obtuse							
Isosceles							
Equilateral							

1. State a conjecture about the sum of the measures of the angles of a triangle.

2. Can you state a conjecture about the exterior angle of a triangle?

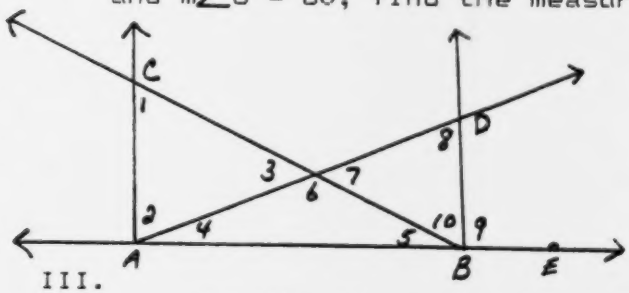
HOMEWORK EXERCISES

I. Use the figure below for exercises 1-7.



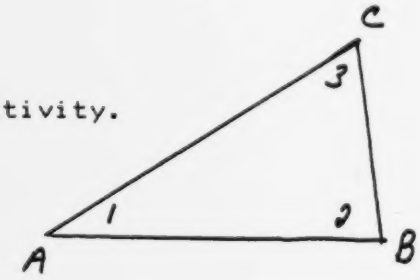
1. $m\angle 1 = 110$; $m\angle 2 = 30$. Find $m\angle 4$.
2. $m\angle 1 = 98$; $m\angle 4 = 124$. Find $m\angle 2$.
3. $m\angle 1 = 88$; $m\angle 2 = 24$. Find $m\angle 3$.
4. $m\angle 2 = 56$; $m\angle 3 = 32$. Find $m\angle 1$.
5. $m\angle 2 = a$; $m\angle 3 = b$. Find $m\angle 1$.
6. $m\angle 1 = 2X+10$; $m\angle 2 = X - 30$; $m\angle 3 = X-4$. Find X.
7. $m\angle 1 = 23X + 3$; $m\angle 2 = 9X + 4$; $m\angle 4 = 35X - 1$. Find X.

II. In the figure below $\overrightarrow{AC} \perp \overleftrightarrow{AB}$ and $\overrightarrow{AC} \perp \overrightarrow{BD}$. If $m\angle 1 = 40$ and $m\angle 8 = 60$, find the measures of 1-10.

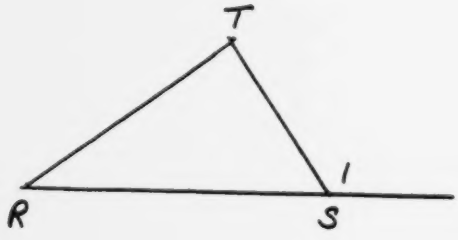


III. Prove. Use the theorems from this lab activity.

1) Given: $\triangle ABC$
   Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180$.



2) Given: $\triangle RST$ with exterior 1.
   Prove: $m\angle 1 = m\angle R + m\angle T$.



HOMEWORK ANSWERS

1. $m\angle 4 = 140$
2. $m\angle 2 = 26$
3. $m\angle 3 = 68$
4. $m\angle 1 = 92$
5. $m\angle 1 = 180 - (a + b)$
6. $x = 51$
7. $x = 8/3$

PART II

$m\angle 1 = 40$

$m\angle 2 = 60$

$m\angle 3 = 80$

$m\angle 4 = 30$

$m\angle 5 = 50$

$m\angle 6 = 100$

$m\angle 7 = 80$

$m\angle 8 = 60$

$m\angle 9 = 90$

$m\angle 10 = 40$

LESSON 3

OBJECTIVE: To show that the segment joining the midpoints of two sides of a triangle has length equal to one-half the length of the third side.

COURSE TITLE: Geometry

SOFTWARE NEEDED: The Geometric Supposer (Sunburst)

SUGGESTED TIME: 1 Lab Period

PREREQUISIE SKILLS: The student should understand midpoint of a line segment and know how to subdivide a segment using the Supposer

ACTIVITY: Discovery Laboratory

MIDSEGMENTS

LAB ACTIVITY:
Use the Supposer to construct triangle ABC having midpoints D, E, and F. Connect these midpoints with line segments and state a conjecture about the relationship these lengths and the lengths of the sides of the original triangle have. Use the REPEAT function with the following triangles to support your data: 1) right 2) acute 3) obtuse 4)isosceles and 5) equilateral.

MEASUREMENTS

Triangle ABC	AB	BC	AC	DE	EF	DF
1)Right						
2)Acute						
3)Obtuse						
4)Isosceles						
5)Equilateral						

State a conjucture drawn from the data shown above:

Can any other conjectures be made ?

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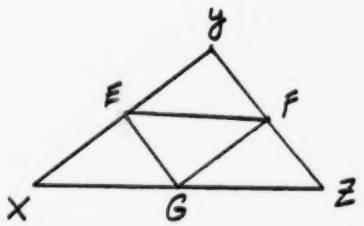
HOMEWORK EXERCISES

For exercises 1-12 , $\triangle XYZ$ is Given with points E, F, G as midpoints of $\overline{XY}$, $\overline{YZ}$, and $\overline{XZ}$ respectively.



1) If XZ = 12.6, EF = ?

2) If EF = 4.2, XZ = ?

3) If FY = 10, EG = ?

4) If XZ = X + 4 and EF = X -2, then X = ?

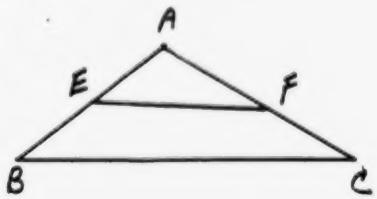
5) If YZ = 3X - 5 and EG = X + 1, then EG = ?

6) If XY = X^2 + 2X + 1 then GF = ?

7) If XZ = X^2 + X + 2 and EF = 2X + 3, then XZ = ?

Given triangle ABC;
E is the midpoint of $\overline{AB}$;

F is the midpoint of $\overline{AC}$



Prove: 2EF = BC

Bonus:
State any additional conjecture(s) in the form of a theorem and verify using a two-column proof.

HOMEWORK ANSWERS

For exercises 1-12 , $\triangle XYZ$ is Given with points E, F, G as midpoints of $\overline{XY}$, $\overline{YZ}$, and $\overline{XZ}$ respectively.

1) If XZ = 12.6, EF = ? 6.3

2) If EF = 4.2, XZ = ? 8.4

3) If FY = 10, EG = ? 10

4) If XZ = X + 4 and EF = X -2, then X = ? 8

5) If YZ = 3X - 5 and EG = X + 1, then EG = ? 7

6) If XY = X^2 + 2X + 1 then GF = ? .5(a^2+2a+1)

7) If XZ = X^2 + X + 2 and EF = 2X + 3, then XZ = ? 22

Given triangle ABC;
E is the midpoint of $\overline{AB}$;

F is the midpoint of $\overline{AC}$

Prove: 2EF = BC

Bonus:
State any additional conjecture(s) in the form of a theorem and verify using a two-column proof.

LESSON 4

OBJECTIVE: To explore the relationships between similar triangles including:

1. The ratios of corresponding sides of similar triangles are equal.
2. The ratio of the perimeters of two similar triangles equals the ratio of corresponding sides.
3. The ratio of the areas of two similar triangles equals the square of the ratio of corresponding sides.

COURSE TITLE: Geometry

SOFTWARE NEEDED: The Geometric Supposer (Sunburst)

SUGGESTED TIME: 1 Lab Period

PREREQUISITE SKILLS: Familiarity with the Supposer. Students should know intuitively that similar triangles have the same shape but may be different in size.

ACTIVITY: Discovery Laboratory

SIMILAR TRIANGLES

LAB ACTIVITY:

Use the Supposer to draw a triangle ABC (any type). Draw triangle ABC in the space provided below Use the Supposer to measure and record the data for triangle ABC. Use the SCALE function to reproduce the same diagram and call the new figure triangle A'B'C'. (The Supposer will label the triangle as triangle ABC. When drawing this triangle on the worksheet let A become A', etc.). Complete the table below.

EXERCISE 1

Draw a diagram of $\triangle ABC$ $\triangle A'B'C'$

Triangle ABC	Triangle A'B'C'
$m\angle ABC =$	$m\angle A'B'C' =$
$m\angle BAC =$	$m\angle A'B'C' =$
$m\angle BCA =$	$m\angle A'B'C' =$
$AB =$	$A'B' =$
$AC =$	$A'C' =$
$BC =$	$B'C' =$
Perimeter of $\triangle ABC =$	Perimeter of $\triangle A'B'C' =$
Area of $\triangle ABC =$	Area of $\triangle A'B'C' =$

EXERCISE 2

Use the Supposer to draw a triangle whose type is different from exercise 1. On triangle ABC construct segment $\overline{DE}$ which is parallel to side $\overline{AC}$ so that point D lies in $\overline{AB}$ and point E lies in $\overline{BC}$. (To construct a parallel select LABEL, RANDOM POINT, ON SEGMENT <AB>, then select DRAW, PARALLEL, THROUGH <D>, PARALLEL TO LINE <AC>, LENGTH DEFINED, and TO INTERNAL SEGMENT <BC>).

Draw the figure:

Triangle ABC	Triangle DBE
$m\angle ABC =$	$m\angle DBE =$
$m\angle BAC =$	$m\angle BDE =$
$m\angle BCA =$	$m\angle BED =$
$AB =$	$DB =$
$AC =$	$DE =$
$BC =$	$BE =$
Perimeter of $\triangle ABC =$	Perimeter of $\triangle DBE =$
Area of $\triangle ABC =$	Area of $\triangle DBE =$

Compare the measures of the corresponding angles in exercises 1 and 2. What conjecture seems valid for the corresponding angles of similar triangles?

Refer to the measurements taken in exercise 1 to make the comparisons shown in the left hand column. Also use the data from exercise 2 to complete the right hand column in a similar manner.

	EXERCISE 1	EXERCISE 2
$\frac{AB}{A'B'} =$		
$\frac{AC}{A'C'} =$		
$\frac{BC}{B'C'} =$		

What conjecture can be made about the corresponding sides of similar triangles?

Describe the ratio of the perimeters of the two similar triangles in exercise 1. Compare this ratio to that of the corresponding sides found above. What seems to be true?

Make the same comparison for exercise 2. Can we still draw the same conclusion? Explain.

What conjecture can you make based on the two previous questions?

Make a conjecture about the areas of similar triangles in comparison to the lengths of corresponding sides. Use the information from exercises 1 and 2 to support your findings.

HOMEWORK EXERCISES

Complete the statement for $\triangle ABC \sim \triangle XYZ$.

1. $\frac{AB}{?} = \frac{AC}{?}$

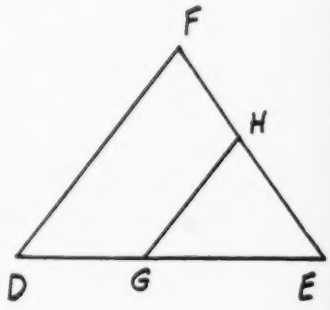
2. $\frac{YZ}{?} = \frac{XZ}{?}$

3. $\frac{AC}{?} = \frac{?}{YZ}$

4. $\angle B \cong$

5. $\angle X \cong$

6. $\angle Z \cong$



Given $\triangle DEF$ with $\overline{GH} \parallel \overline{DF}$. Complete the table.

	DG	GE	FH	EH	$\frac{GH}{DF}$
7.	3	5	4		
8.	2		3	5	
9.	3X	2Y		4X	
10.	$4\sqrt{3}$	$2\sqrt{3}$	$3\sqrt{2}$		

Given $\triangle ABC \sim \triangle DEF$. Complete the table.

	$\frac{AB}{DE}$	$\frac{\text{Perimeter ABC}}{\text{Perimeter DEF}}$	$\frac{\text{Area ABC}}{\text{Area DEF}}$
1.	2:3	?	?
2.	?	3:4	?
3.	?	?	64:49
4.	?	X:Y	?

PIMMS

HOMEWORK ANSWERS

Complete the statement for $\triangle ABC \sim \triangle XYZ$.

1. $\frac{AB}{XY} = \frac{AC}{XZ}$

2. $\frac{YZ}{BC} = \frac{XZ}{AC}$

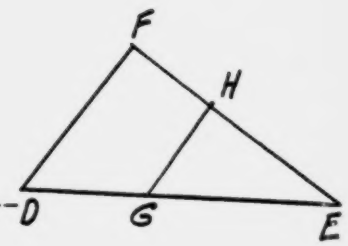
3. $\frac{AC}{XZ} = \frac{BC}{YZ}$

4. $\angle B \cong \angle Y$

5. $\angle X \cong \angle A$

6. $\angle Z \cong \angle C$

Given $\triangle DEF$ with $\overline{GH} \parallel \overline{DF}$. Complete the table.



	DG	GE	FH	EH	$\frac{GH}{DF}$
7.	3	5	4	20/3	5/8
8.	2	10/3	3	5	5/8
9.	3X	2Y	6X^2/Y	4X	2Y/(3X+2Y)
10.	$4\sqrt{3}$	$2\sqrt{3}$	$3\sqrt{2}$	$3\sqrt{2}/2$	1/3

Given $\triangle ABC \sim \triangle DEF$. Complete the table.

	$\frac{AB}{DE}$	$\frac{\text{Perimeter ABC}}{\text{Perimeter DEF}}$	$\frac{\text{Area ABC}}{\text{Area DEF}}$
1.	2:3	2:3	4:9
2.	3:4	3:4	9:16
3.	8:7	8:7	64:49
4.	X:Y	X:Y	X^2:Y^2

PIMMS

LESSON 5

OBJECTIVE: To discover that the sum of the lengths of the three perpendiculars from a random point in the interior of an equilateral triangle to the sides is equal to altitude of the triangle.

COURSE TITLE: Geometry

SOFTWARE NEEDED: The Geometric Supposer (Sunburst)

SUGGESTED TIME: 1 Lab Period

PREREQUISITE: The students should know what an altitude is and should be able to use the Supposer to draw an altitude, locate a random point in the interior of a triangle, draw perpendiculars from a point to a segment, and make measurements.

ACTIVITY: Discovery Laboratory

DIRECTIONS:

To draw a perpendicular segment from a random point in the interior of a triangle to a side of the triangle, give the following responses to the successive choices offered within the program.

Step	CHOICE	RESPONSE
1	LABEL	
2	RANDOM POINT	
3	INSIDE TRIANGLE	ABC
4	DRAW	
5	PERPENDICULAR	
6	THROUGH POINT	D
7	TO LINE	AC
8	LENGTH DEFINED BY	
9	TO INTERSECT SEGMENT(S)	AC
10	(a prompting message will appear)	Respond to the prompt by returning to Step 4 until all perpendiculars are drawn. Then hit RETURN to return to the main MENU.

PERPENDICULARS FROM A RANDOM POINT

LAB ACTIVITY:
Use the Supposer to construct an equilateral triangle ABC and locate a random point D in the interior of the triangle. Draw segments from D perpendicular to the sides of the triangle at points E, F, and G. Also, draw an altitude from any vertex. Measure lengths as indicated in the chart below and record the data. Repeat the procedure for 5 equilateral triangles and make a conjecture based on the recorded data. Use the reverse side to draw the last 3 triangles.

Draw a diagram of the construction and label all parts.

1.

2.

EQUILATERAL TRIANGLE	DE	DF	DG	Altitude
Triangle 1				
Triangle 2				
Triangle 3				
Triangle 4				
Triangle 5				

What conjecture can you make based on the above data?

Would this conjecture be valid using a triangle that is not equilateral ?

Support your answer with examples using the Supposer.

	Type of Triangle	DE	DF	DG	Altitude
1.					
2.					
3.					

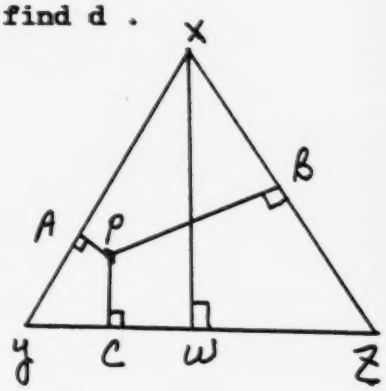
PIMMS

State the conjecture which was made in this lab activity in the form of a theorem, draw a diagram, and prove that the sum of the lengths of the three perpendiculars from a random point in the interior of an equilateral triangle to the sides is equal to the altitude of the triangle.

HOMEWORK EXERCISES

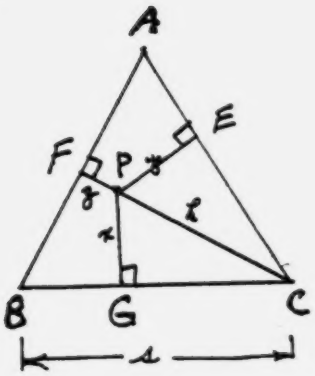
Given equilateral triangle XYZ with altitude XW, point P in the interior of the triangle, PA $\perp$ XY, PB $\perp$ XZ, PC $\perp$ YZ, Find the indicated lengths.

1. XW = 4, PA = 1.2, PB = .9, PC = ?
2. PA = 3.2, PB = 1.9, PC = 2.6, XW = ?
3. XW = 8, PB = b, PC = c, PA = ?
4. XW = 4d, PA = d + 3, PB = 2d - 1, PC = 3, find d .



PIMMS

State the conjecture which was made in this lab activity in the form of a theorem, draw a diagram, and prove that the sum of the lengths of the three perpendiculars from a random point in the interior of an equilateral triangle to the sides is equal to the altitude of the triangle.



Given equilateral $\triangle ABC$ with altitude $\overline{CF}$, $\overline{DE} \perp \overline{AC}$, $\overline{DF} \perp \overline{AB}$, $\overline{DG} \perp \overline{BC}$, prove that $DF + DE + DG = CF$.

Proof: $K_{ABC} = K_{ABD} + K_{BCD} + K_{ACD}$

$$\frac{1}{2} AB \cdot CF = \frac{1}{2} AB \cdot FD + \frac{1}{2} BC \cdot DG + \frac{1}{2} AC \cdot DE$$

since $\triangle ABC$ is equilateral, $AB = BC = AC$. Use s for this length.

$$\frac{1}{2} s \cdot CF = \frac{1}{2} s \cdot FD + \frac{1}{2} s \cdot DG + \frac{1}{2} s \cdot DE$$

$$CF = FD + DG + DE$$

HOMEWORK EXERCISES

Given equilateral triangle XYZ with altitude XW, point P in the interior of the triangle, $PA \perp XY$, $PB \perp XZ$, $PC \perp YZ$, find the indicated lengths.

1. XW = 4, PA = 1.2, PB = .9, PC = ? 1.9
2. PA = 3.2, PB = 1.9, PC = 2.6, XW = ? 7.7
3. XW = 8, PB = b, PC = c, PA = ? 8 - (b + c)
4. XW = 4d, PA = d + 3, PB = 2d - 1, PC = 3, find d . 5

TITLE : GRAPHING EQUATIONS
PUBLISHER : CONDUIT EDUCATIONAL SOFTWARE
ADDRESS : THE UNIVERSITY OF IOWA
OAKDALE CAMPUS
IOWA CITY, IOWA 52242
(also available from Sunburst)

PRICE : $99.00 (five-diskette lab pack plus instruction manual)

PIMMS Rating : EXCELLENT

PROGRAM IS APPROPRIATE FOR

ABILITY GROUP	:	MIDDLE, HIGH
COURSE	:	ALGEBRA I, II TRIGONOMETRY
FORMAT	:	CLASS/DEMO, DISCOVERY/LABORATORY, DRILL AND PRACTICE, TUTORIAL, GAME
CHARACTERISTICS	:	DISK DEPENDENT TO MOVE BETWEEN THE FOUR MAJOR COMPONENTS BUT NOT DISK DEPENDENT TO WORK WITHIN A SINGLE COMPONENT; GRAPHING UTILITY; TWO GAME FORMATS; DOCUMENTATION; MAY BE SCREEN DUMPED
COMPUTER	:	APPLE II+, E, C
ADDITIONAL PERIPHERALS	:	LARGE MONITOR FOR CLASSROOM DEMONSTRATIONS

GRAPHING EQUATIONS by Sharon Dugdale and David Kibbey offers a means of supplementing and enhancing the teaching of elementary functions and the conic sections. It is a very versatile software package which includes a powerful graphing utility, two "fun" and worthwhile game formats, and a computer-controlled graphing component requiring user identification of the graphed function.

The software is completely menu-driven with very good screen instructions directing the user. The content ranges from simple linear graphing to the graphing of the trigonometric functions. Within each of the content areas, the user is able to control the level of difficulty. This ease of use combined with controllable content (type and difficulty level) make GRAPHING EQUATIONS adaptable to a variety of instructional formats.

PIMMS

GRAPHING EQUATIONS has four major components:

In Section 1, Linear and Quadratic Graphs, the student is offered practice in five different graph areas: lines, circles, ellipses, parabolas, hyperbolas. The computer presents a graphical image within the selected topic. The student's task is to write an appropriate equation for the given graph. When the correct equation is given, the computer moves on to a more difficult level. (There are up to twenty levels of difficulty available).

Green Globs, Section 2, is a single-user competitive game. In this fun and challenging game the computer presents a series of green dots randomly plotted on a rectangular grid. The student enters functional equations to 'shoot' as many globs as possible per equation. Points are earned per glob shot. Scoring records may be written to the disk and any game previously played may be replayed.

The object of Tracker, Section 3, is to locate and identify graphs by using horizontal and vertical probes. The hidden graphs may be any of five types--lines, circles, ellipses, parabolas, hyperbolas--with two different graph types 'hidden' for each game. Tracker is the most challenging component of this software, requiring skill and patience.

The Equation Plotter in Section 4 is a general-purpose graphing utility. Using either a rectangular or square grid the user may graph conic sections and general functions, including the square, absolute value, logarithmic and exponential. The x and y bounds may be adjusted for either of the grid types and trigonometric functions.

The sample lessons contained within this packet are designed primarily for use in a lab setting. With minor adjustments the material is equally usable in a classroom setting.

Appendix A contains detailed information on the booting, accessing, and use of Graphing Equations.

REVIEWING LINEAR EQUATIONS
LESSON 1

OBJECTIVE: To review the relationship between the equation of a line (in both point-slope and slope-intercept forms) and its graph.

To graph a line given its equation in either point-slope or slope-intercept form.

COURSE TITLE: Algebra II

TITLE OF SOFTWARE PACKAGE NEEDED FOR THE LESSON:
Graphing Equations by Conduit (or any graphing utility which allows entry of linear equations in alternate forms).

TYPE OF ACTIVITY: Classroom/Lab

SUGGESTED TIMELINE: 1-2 periods

PREREQUISITE SKILLS:
The student should be:

- familiar with the terminology of linear functions: slope, x- and y-intercepts, point, quadrants, horizontal, vertical.
- able to graph linear functions.
- familiar with the 'Equation Plotter' portion of Graphing Equations (or a comparable graphing utility).

PIMMS

REVIEWING LINEAR EQUATIONS
LESSON 1

LABORATORY EXERCISES

Use the 'Equation Plotter' option of Graphing Equations:

1. Graph the equation. Then copy to graph paper and label.

a) $y-2 = 3(x+1)$ b) $y = 2x-2$ c) $y = 3$

d) $y+1 = -2(x-3)$ e) $x = -2$ f) $y = (1/2)x+3$

g) $y-0 = 1/3(x+6)$ h) $y-3 = 2x$ i) $y = -6$

j) $x = 0$ k) $y = -x$ l) $y = (-1/5)x+1$

2. Use your own graphs and the 'Equation Plotter'. Answer the question and support your answer with evidence.

a) Does the value of m affect the location of the x or y intercept?

b) What does m tell us about the graph of an equation?

c) For linear graphs in quadrants I and III, m is always

d) For linear graphs in quadrants II and IV, m is always

e) If the slope is 0 and the y-intercept is not 0 then the graph must lie in quadrants _____ and _____ or in quadrants _____ and _____.

f) If there is no slope and the x-intercept is not 0 then the graph of the equation must lie in quadrants _____ and _____ or in quadrants _____ and _____.

g) When m is positive, what happens to the graph as m gets larger?

h) When m is negative, what happens to the graph as m becomes more negative?

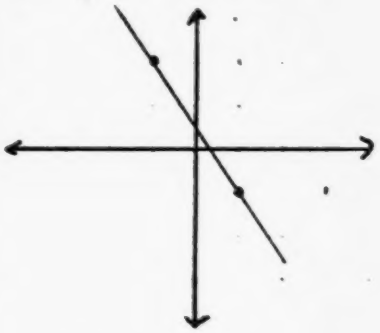
i) For positive values of b, is the y-intercept located above or below the x axis?

j) For negative values of b, is the y-intercept located above or below the x axis?

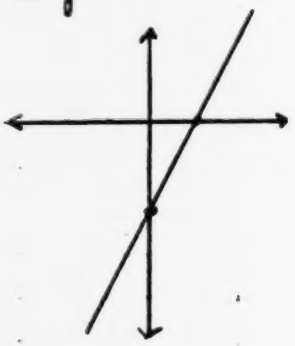
REVIEWING LINEAR EQUATIONS
LESSON I
ANSWERS TO LABORATORY EXERCISES

1.

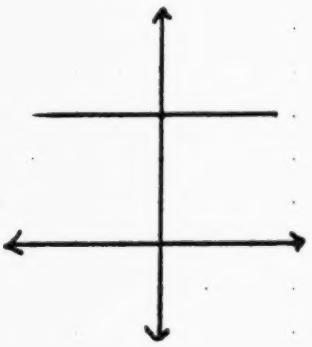
a) $y-2=3(x+1)$



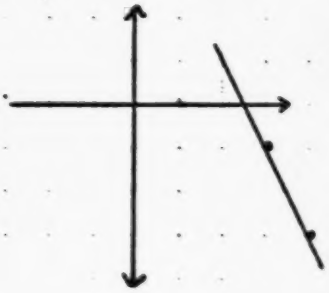
b) $y=2x-2$



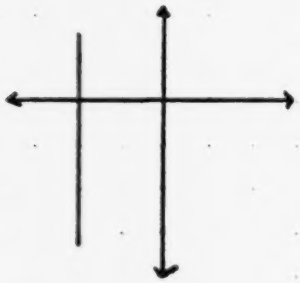
c) $y=3$



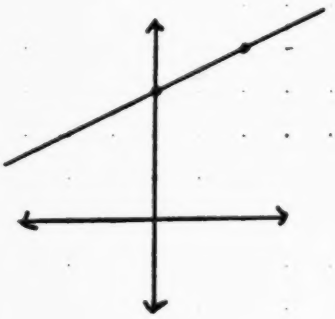
d) $y+1=-2(x-3)$



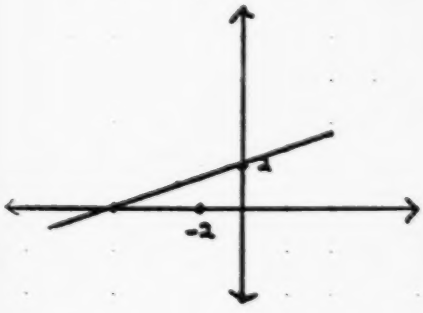
e) $x=-2$



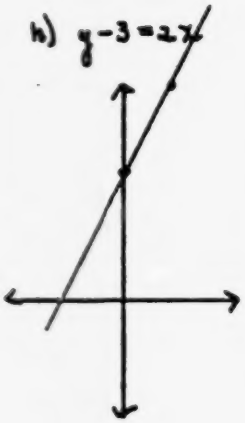
f) $y=(1/2)x+3$



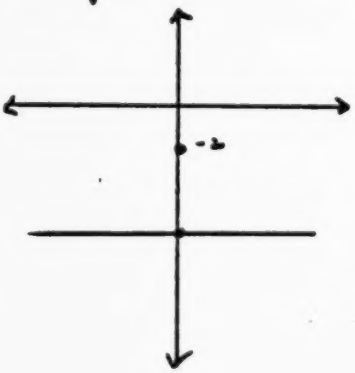
g) $y-0=1/3(x+6)$



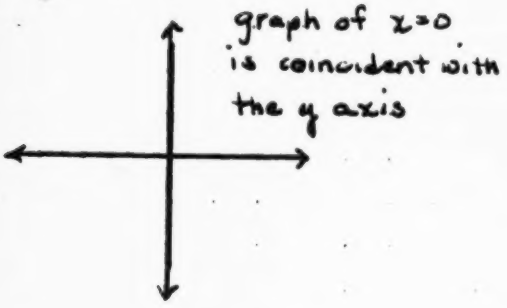
h) $y-3=2x$



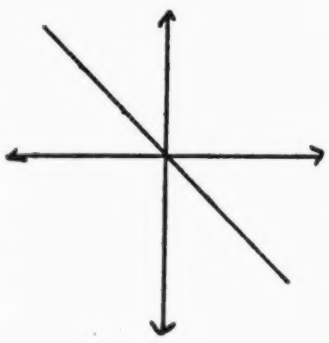
i) $y=-6$



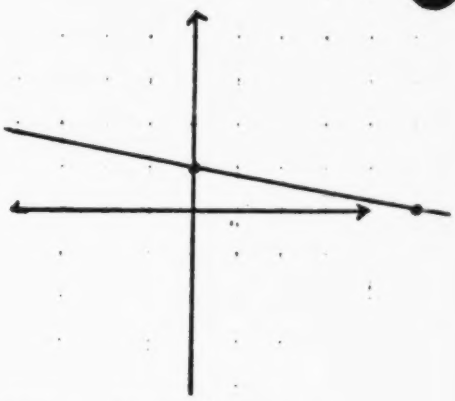
j) $x=0$



k) $y=-x$



l) $y=(-\frac{1}{3})x+1$



2. Use your own graphs and the 'Equation Plotter'. Answer the question and support your answer with evidence.

a) Does the value of m affect the location of the x or y intercept? no
no

b) What does m tell us about the graph of an equation? its steepness

c) For linear graphs in quadrants I and III, m is always positive

d) For linear graphs in quadrants II and IV, m is always negative

e) If the slope is 0 and the y intercept is not 0 then the graph must lie in quadrants I and II or in quadrants III and IV.

f) If there is no slope and the x intercept is not 0 then the graph of the equation must lie in quadrants I and IV or in quadrants II and III.

g) When m is positive, what happens to the graph as m gets larger?
the graph becomes steeper

h) When m is negative, what happens to the graph as m becomes more negative? the graph becomes steeper

i) For positive values of b, is the y-intercept located above or below the x axis? above

j) For negative values of b, is the y-intercept located above or below the x axis? below

REVIEWING LINEAR EQUATIONS
HOMEWORK EXERCISES
LESSON 1

1. Graph and label.

a) $y = -2x+1$ b) $y+2 = 1(x-3)$ c) $x = -4$

d) $y-1 = -1/2(x+3)$ e) $y = 2x$ f) $y-2 = 2(x-1)$

g) $y = x$ h) $x = 2$ i) $y = (1/3)x-3$

j) $y = -3x+6$ k) $y+4 = (1/4)x$ l) $y = 5$

2.

a) What does the graph look like if $m = 0$?

b) What does the graph look like if the slope is undefined?

c) Does changing the value of b have any effect on $y = mx+b$ graphs? If so, how does the graph change?

d) How does changing the value of m affect the graphs of linear equations?

e) How does the slope-intercept form tell what the y-intercept is?

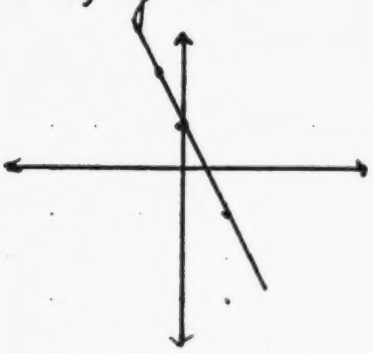
f) The x-intercept may be determined by examining the equation of a line in $y = mx+b$ form. Illustrate this process using $y = 3x-2$ and $y = -2x-1$.

PIMMS

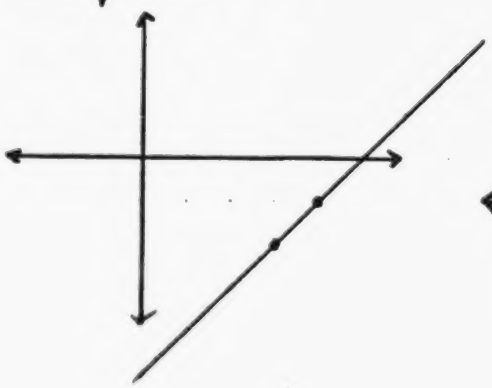
REVIEWING LINEAR EQUATIONS
LESSON 1
ANSWERS TO HOMEWORK EXERCISES

1.

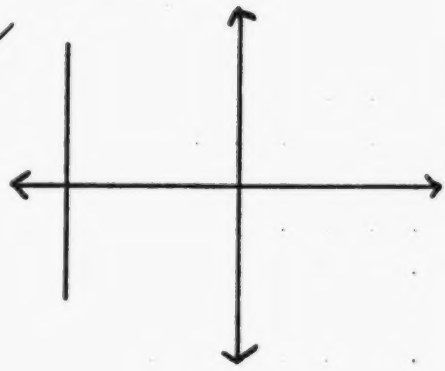
a) $y = -2x + 1$



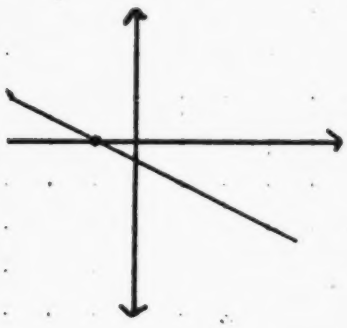
b) $y + 2 = 1(x - 3)$



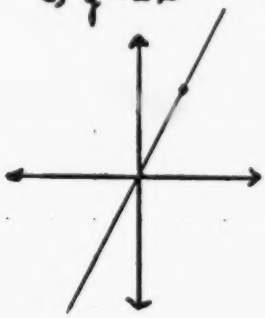
c) $x = -4$



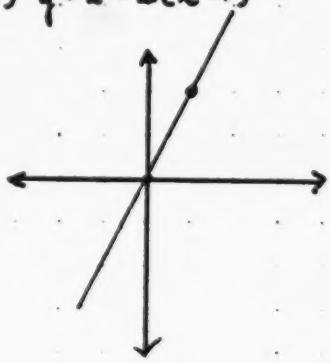
d) $y - 1 = -\frac{1}{2}(x + 3)$



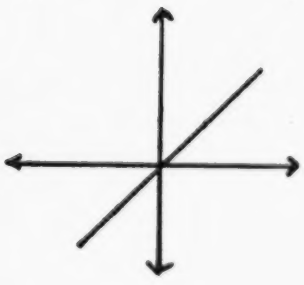
e) $y = 2x$



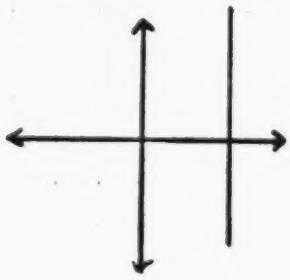
f) $y - 2 = 2(x - 1)$



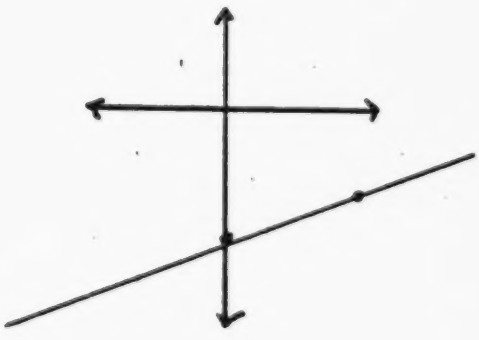
g) $y = x$



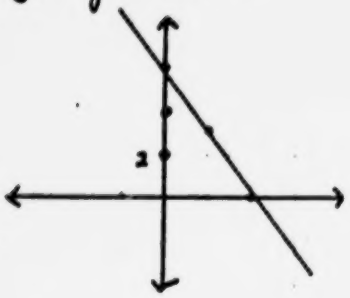
h) $x = 2$



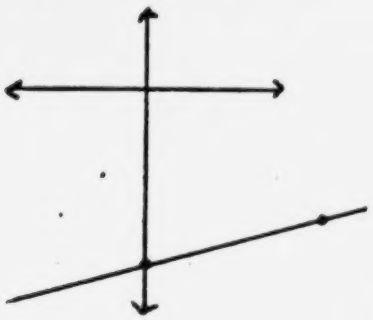
i) $y = (\frac{1}{3})x - 3$



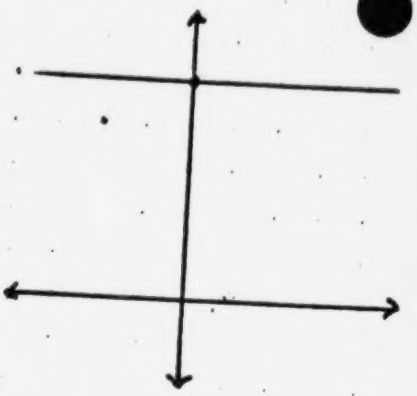
j) $y = -3x+6$



k) $y+4 = (1/4)x$



l) $y = 5$



2.

a) Wnat does the graph look like if m = 0?

The graph is a horizontal line.

b) What does the graph look like if the slope is undefined?

The graph is a vertical line.

c) Does changing the value of b have any effect on y = mx+b graphs? If so, how does the graph change?

Changing b gives a series of parallel lines that intersect the y-axis at various points.

d) How does changing the value of m affect the graphs of linear equations?

As m is increased the steepness of the graph is increased.

e) How does the slope-intercept form tell what the y-intercept is?

For $y = mx+b$, b gives the y-intercept of the graph.

f) The x-intercept may be determined by examining the equation of a line in y = mx+b form. Illustrate this process using y = 3x-2 and y = -2x-1.

For $y = mx+b$, the x-intercept is given when $y = 0$.

$$y = mx + b$$

PIMMS

$$0 = mx + b$$

$$x = -\frac{b}{m}$$

For $y = 3x-2$, the x-intercept is at $x = \frac{2}{3}$.

For $y = -2x-1$, the x-intercept is at $x = -\frac{1}{2}$.

REVIEWING LINEAR EQUATIONS
LESSON 2

OBJECTIVE: To review the relationship between the graph of a line and its equation.

To write the equation of a linear function given its graph.

COURSE TITLE: Algebra II

TITLE OF SOFTWARE PACKAGE NEEDED FOR THE LESSON:
Graphing Equations by Conduit (or any graphing utility which allows entry of linear equations in alternate forms).

TYPE OF ACTIVITY: Classroom/Lab

SUGGESTED TIMELINE: 1-2 periods

PREREQUISITE SKILLS:
The student should be:

- familiar with the vocabulary of linear functions: slope, x- and y-intercepts, point, quadrants, horizontal, vertical
- able to graph linear functions
- able to recognize and interpret linear equations presented in either slope-intercept or point-slope forms
- familiar with the 'Equation Plotter' portion of Graphing Equations (or a comparable graphing utility)

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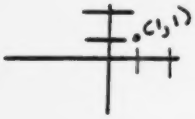
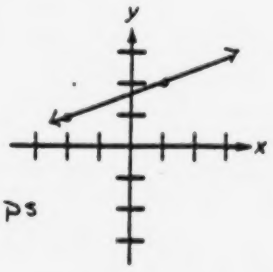
REVIEWING LINEAR EQUATIONS
LESSON 2

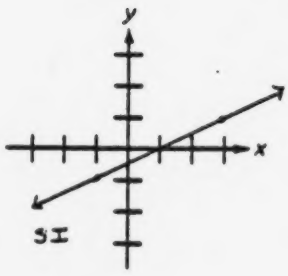
LABORATORY EXERCISES

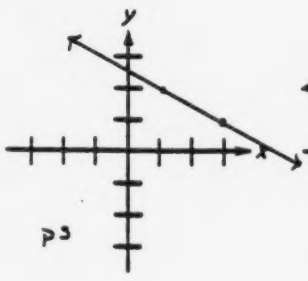
1. Examine the graph. Write its equation in the form indicated (PS = point-slope form; SI = slope-intercept form). Then use 'Equation Plotter', option 4 of Graphing Equations, to check.

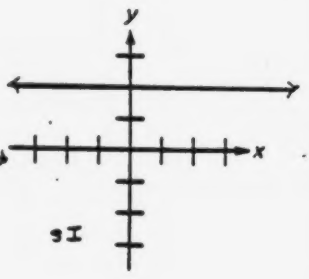
PS = point-slope
SI = Slope-intercept



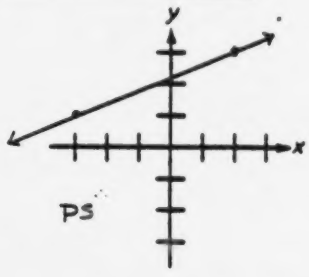








a)



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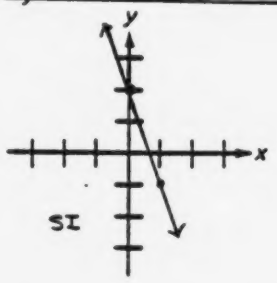

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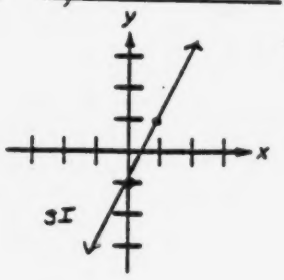

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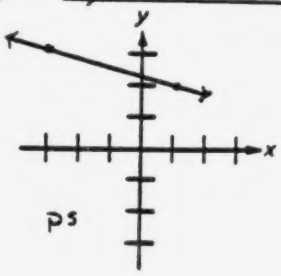

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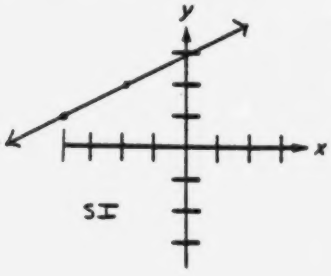

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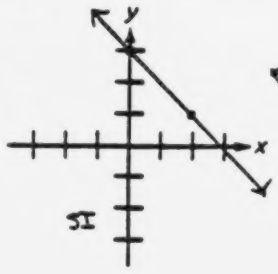

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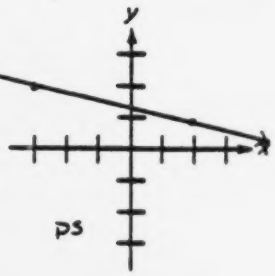

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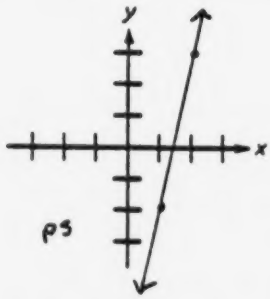

i)

j)

k)

l)

2. Give your response for each item. Use your own graphs and the Equation Plotter to check.

a) Give the y-intercept for the graph of the equation $y = 3x+1$.

b) Give the y-intercept for any graph in the form $y = mx+b$.

c) When we graph $y = 2x-1$ and $y-3 = 2(x+4)$, the lines will be _____.

d) Write an equation whose graph is a horizontal line.

e) Write an equation whose graph is a vertical line containing $(2,0)$.

f) Write an equation whose graph is parallel to and between the graphs of $y = 3x-4$ and $y = 3x+1$.

g) Write an equation whose graph is a line parallel to the graph of $y-3 = 2(x-1)$ but passes through the point $(2,5)$.

h) Graph $y = 3x+1$ and $y = -1/3x+2$ on the same axes. Examine the point at which these lines intersect. What appears to be true about the angle created at this point of intersection?

i) Perpendicular lines have slopes that are _____.

j) Write an equation whose graph is perpendicular to the graph of $y = 2x-1$.

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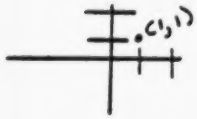
ANSWER KEY

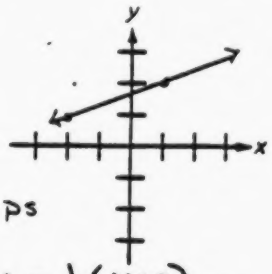
REVIEWING LINEAR EQUATIONS
LESSON 2

LABORATORY EXERCISES

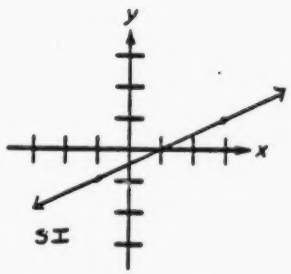
1. Examine the graph. Write its equation in the form indicated (PS = point-slope form; SI = slope-intercept form). Then use 'Equation Plotter', option 4 of Graphing Equations, to check.

PS = point-slope
SI = slope-intercept

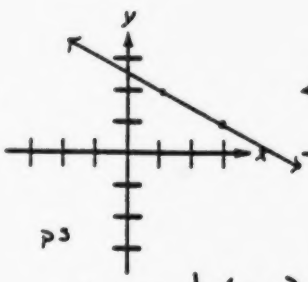




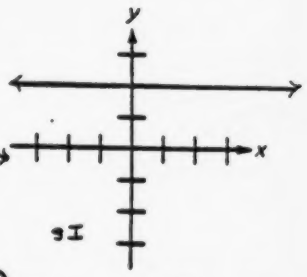
a) $y-1=\frac{1}{3}(x+2)$ or $y-2=\frac{1}{3}(x-1)$



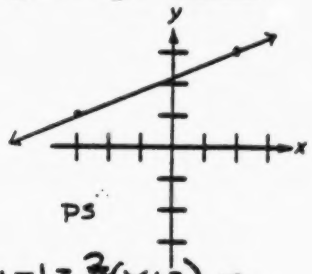
b) $y=\frac{1}{2}x-\frac{1}{2}$



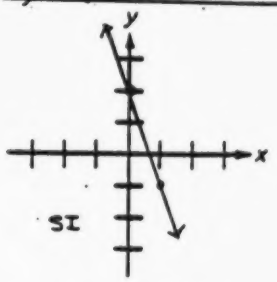
c) $y-1=-\frac{1}{2}(x-3)$ or $y-2=-\frac{1}{2}(x-1)$



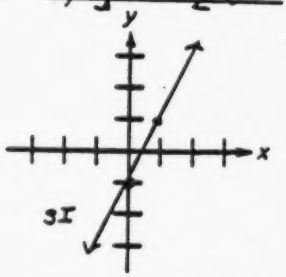
d) $y=2$ or $y=0x+2$



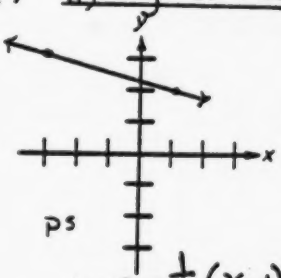
e) $y-1=\frac{3}{5}(x+3)$ or $y-3=\frac{3}{5}(x-2)$



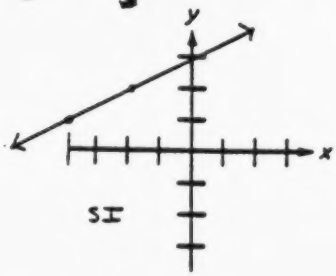
f) $y=-3x+2$



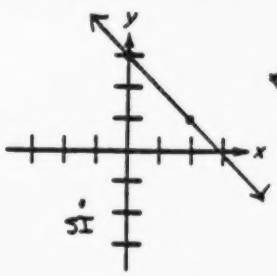
g) $y=2x-1$



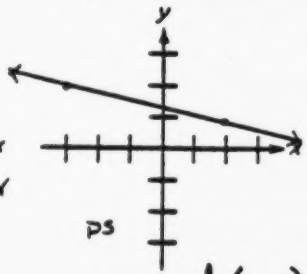
h) $y-2=\frac{1}{4}(x-1)$ or $y-3=\frac{1}{4}(x+3)$



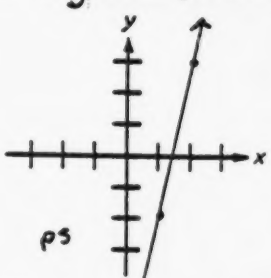
i) $y=\frac{1}{2}x+3$



j) $y=x+3$



k) $y-1=-\frac{1}{5}(x-2)$ or $y-2=-\frac{1}{5}(x+3)$



l) $y+2=5(x-1)$ or $y-3=5(x-2)$

2. Give your response for each item. Use your own graphs and the Equation Plotter to check.

a) Give the y-intercept for the graph of the equation $y = 3x+1$.

$(1,0)$

b) Give the y-intercept for any graph in the form $y = mx+b$.

$(b,0)$

c) When we graph $y = 2x-1$ and $y-3 = 2(x+4)$, the lines will be parallel.

d) Write an equation whose graph is a horizontal line.

$y=k$ for $k \in R$

e) Write an equation whose graph is a vertical line containing (2,0).

$x = 2$

f) Write an equation whose graph is parallel to and between the graphs of $y = 3x-4$ and $y = 3x+1$.

$y = 3x+b$, $-4<b<1$

g) Write an equation whose graph is a line parallel to the graph of $y-3 = 2(x-1)$ but passes through the point (2,5).

$y-5=2(x-2)$

h) Graph $y = 3x+1$ and $y = -1/3x+2$ on the same axes. Examine the point at which these lines intersect. What appears to be true about the angle created at this point of intersection?

The angle appears to have measure 90°.

i) Perpendicular lines have slopes that are "negative reciprocals" — i.e., the product of their slopes is -1.

j) Write an equation whose graph is perpendicular to the graph of $y = 2x-1$.

$y = -\frac{1}{2}x + b$, $b \in R$

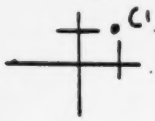
PIMMS

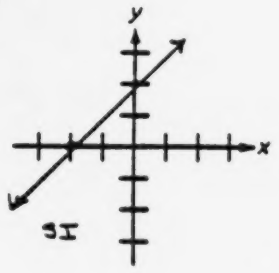
REVIEWING LINEAR EQUATIONS
HOMEWORK EXERCISES
LESSON 2

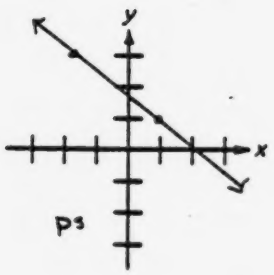
Give an equation for the graph in the form indicated (PS means point-slope form, and SI means slope-intercept form). If the equation for a particular graph cannot be given in one of the forms indicated, give that equation in the constant form.

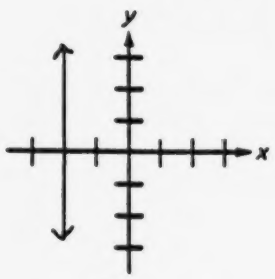
1.

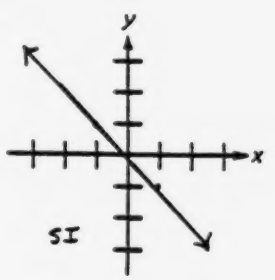
PS = point slope
SI = slope intercept

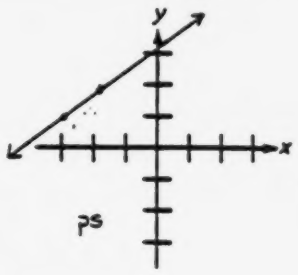


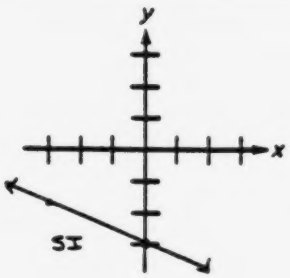
a) 

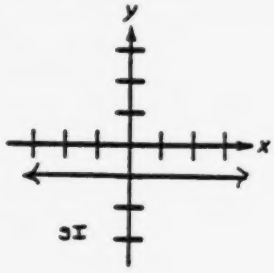
b) 

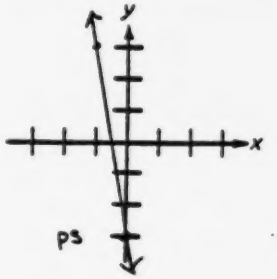
c) 

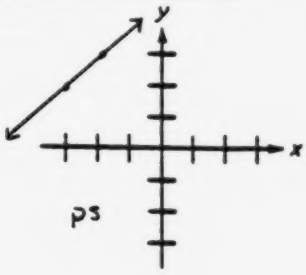
d) 

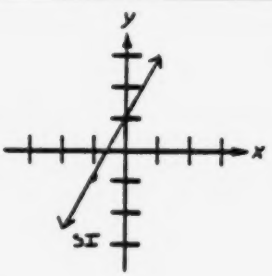
e) 

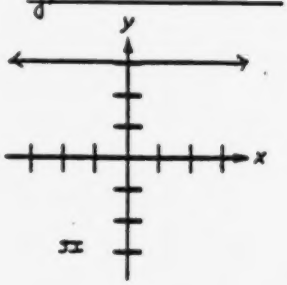
f) 

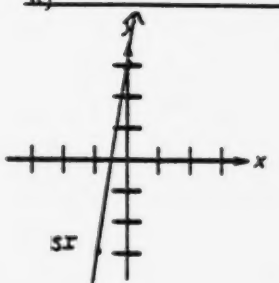
g) 

h) 

i) 

j) 

k) 

l) 

PIMMS

2. Give your response for each item. Graph where indicated.

a) Give the slope of $y-3 = -2(x-1)$.

b) Name a point contained in a) above.

c) Give the slope of the graph for any equation given by $y-y_1 = m(x-x_1)$.

d) Write the equation of a line containing $(2,-3)$ with slope -1. Graph.

e) Does the graph of $y = 3x+2$ contain $(0,2)$? Graph.

f) Are the graphs of $y-5 = 1/2(x+3)$ and $y = 1/2x-2$ parallel? Graph.

g) Draw the graph of a line parallel to $y = x-1$. Give its equation.

h) What part of the equations of two or more lines will tell you whether the lines are parallel?

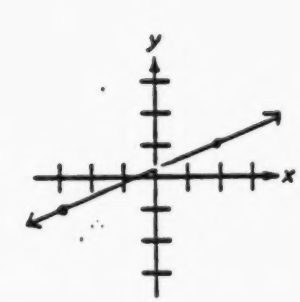
i) Give the relationship of the graphs of $(y-2) = 3(x+1)$ and $y = -1/3x$. Graph.

j) Write the equation of a line whose graph is perpendicular to the graph of $y+2 = 1/2(x+1)$. Graph.

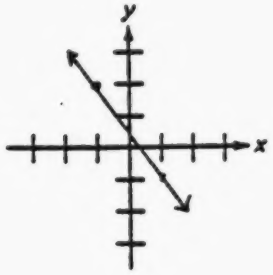
k) How can you tell by examining linear equations whether their graphs will be perpendicular? Give an example other than ones used in these exercises of two such linear equations.

3. Match each graph with its equation. (Note that there are more equations than graphs.)

1) $y-4 = 2/3(x-4)$
2) $x = -2$
3) $y-2 = 3/2(x+2)$
4) $y+1 = 2/5(x+3)$
5) $y-2 = -3/2(x+1)$
6) $y = 5/2x$
7) $y+2 = -3/2(x-1)$
8) $y = 5x+3$
9) $y = 2/3x+1$
10) $y+1 = -4(x-3)$
11) $y+5 = 3(x-2)$
12) $y = 2$
13) $x-3 = -1/4(y+1)$
14) $y-4 = 3/2(x-2)$



a) _____



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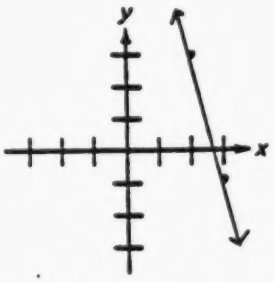
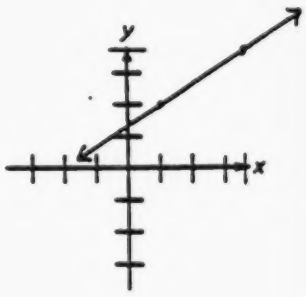
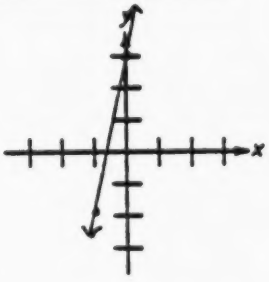


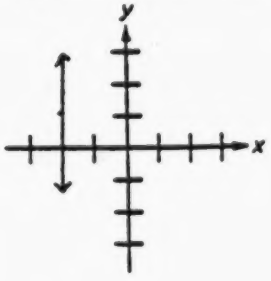
c) _____



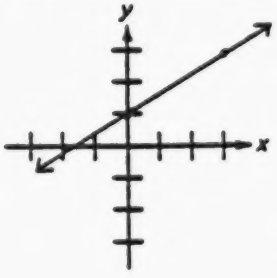
d) _____



e) _____



f) _____



g) _____

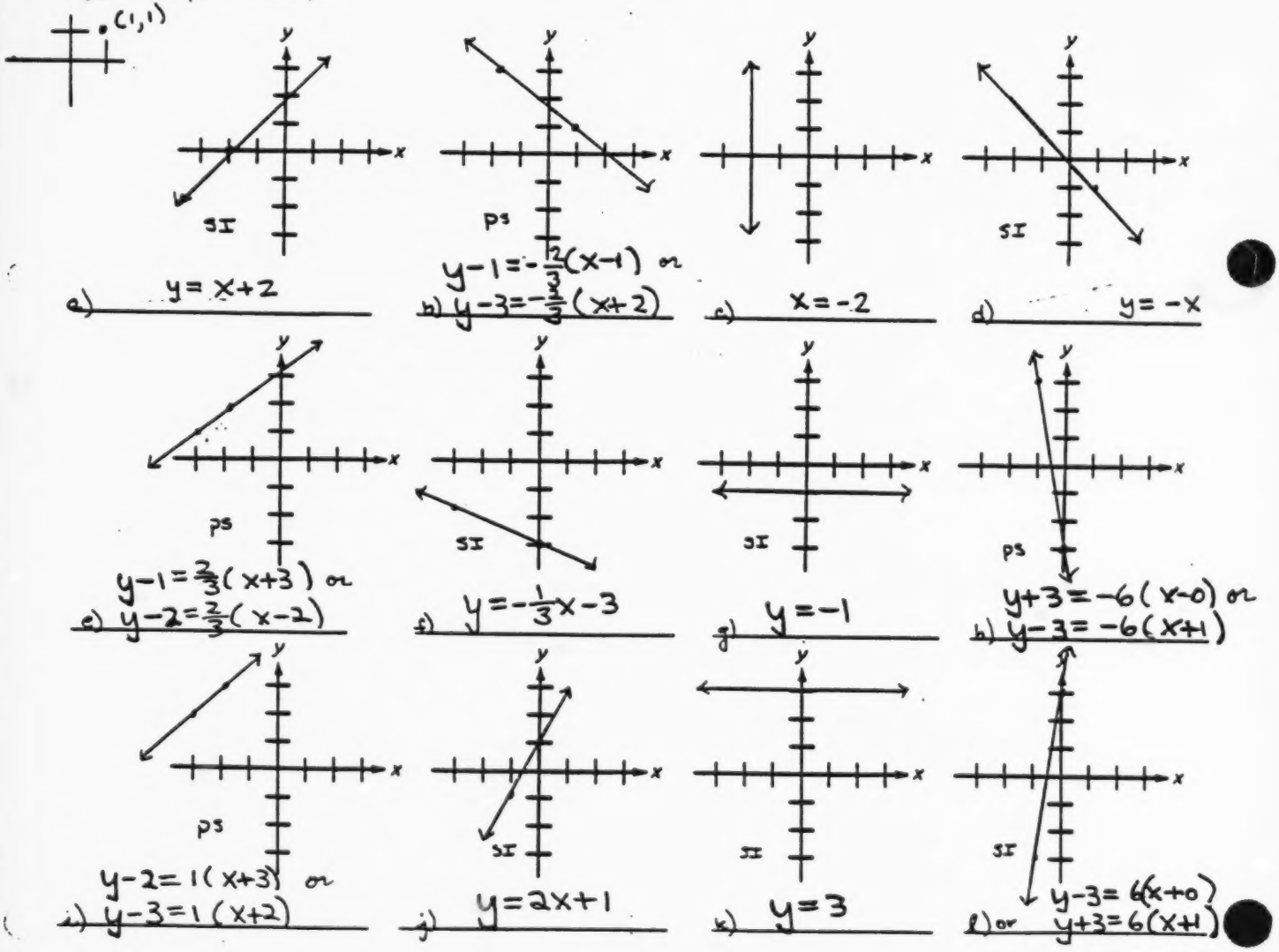
ANSWER KEY

REVIEWING LINEAR EQUATIONS
HOMEWORK EXERCISES
LESSON 2

Give an equation for the graph in the form indicated (PS means point-slope form, and SI means slope-intercept form). If the equation for a particular graph cannot be given in one of the forms indicated, give that equation in the constant form.

1.

ps = point slope
si = slope intercept



a) $y = x+2$

b) $y-1=-\frac{2}{3}(x-1)$ or $y-3=-\frac{2}{3}(x+2)$

c) $x=-2$

d) $y=-x$

e) $y-1=\frac{2}{3}(x+3)$ or $y-2=\frac{2}{3}(x-2)$

f) $y=-\frac{1}{3}x-3$

g) $y=-1$

h) $y+3=-6(x-0)$ or $y-3=-6(x+1)$

i) $y-2=1(x+3)$ or $y-3=1(x+2)$

j) $y=2x+1$

k) $y=3$

l) $y-3=6(x+0)$ or $y+3=6(x+1)$

2. Give your response for each item. Graph where indicated.

a) Give the slope of $y-3 = -2(x-1)$.

$m = -2$

b) Name a point contained in a) above.

$(1,3)$

c) Give the slope of the graph for any equation given by $y-y_1 = m(x-x_1)$.

m

d) Write the equation of a line containing $(2,-3)$ with slope -1. Graph.

$y+3 = -(x-2)$

e) Does the graph of $y = 3x+2$ contain $(0,2)$? Graph.

$2 \stackrel{?}{=} 3(0)+2$

$2 \stackrel{\checkmark}{=} 2 \quad \Rightarrow$ The graph contains $(0,2)$.

f) Are the graphs of $y-5 = 1/2(x+3)$ and $y = 1/2x-2$ parallel? Graph.

$m_1 \stackrel{?}{=} m_2$

$\frac{1}{2} \stackrel{\checkmark}{=} \frac{1}{2} \quad \Rightarrow$ the graphs are parallel lines.

g) Draw the graph of a line parallel to $y = x-1$. Give its equation.

$y = x + b, \; b \neq 1$

h) What part of the equations of two or more lines will tell you whether the lines are parallel?

When the equations for the lines are in $y=mx+b$ form, equations having equal values for m will give graphs that are parallel lines.

i) Give the relationship of the graphs of $(y-2) = 3(x+1)$ and $y = -1/3x$. Graph.

The graphs of the equations are perpendicular lines.

j) Write an equation for a line whose graph is perpendicular to the graph of $y+2 = 1/2(x+1)$. Graph.

$y-y_1 = -2(x-x_1)$

PIMMS

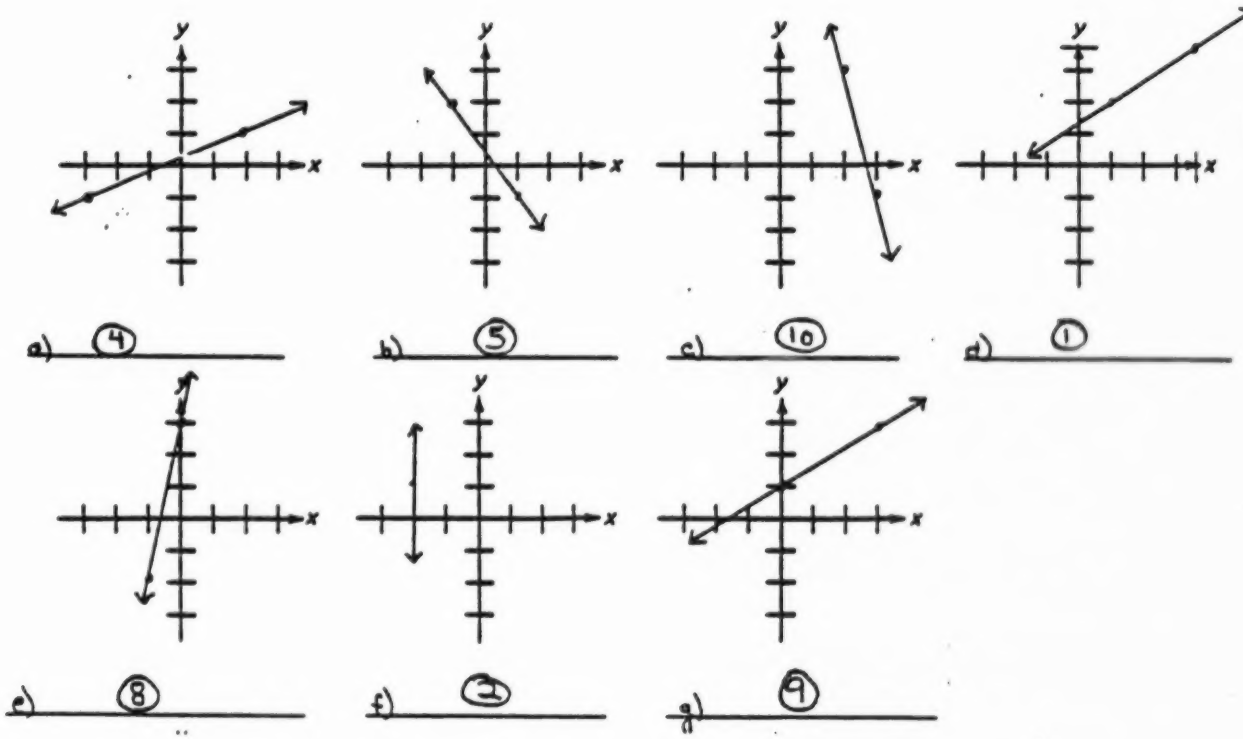
k) How can you tell by examining linear equations whether their graphs will be perpendicular? Give an example other than ones used in these exercises of two such linear equations.

The product of the slopes of the two graphs must equal -1.

$y = \frac{2}{3}x + 1$ and $y = -\frac{3}{2}x - 2$

3. Match each graph with its equation. (Note that there are more equations than graphs.)

1) $y-4 = 2/3(x-4)$
2) $x = -2$
3) $y-2 = 3/2(x+2)$
4) $y+1 = 2/5(x+3)$
5) $y-2 = -3/2(x+1)$
6) $y = 5/2x$
7) $y+2 = -3/2(x-1)$
8) $y = 5x+3$
9) $y = 2/3x+1$
10) $y+1 = -4(x-3)$
11) $y+5 = 3(x-2)$
12) $y = 2$
13) $x-3 = -1/4(y+1)$
14) $y-4 = 3/2(x-2)$



REVIEWING LINEAR EQUATIONS
LESSON 3

OBJECTIVE: To review the relationship between given information about linear equations and their graphs.

To write and graph linear functions given information regarding the slope and y-intercept or the slope and a point or two points.

COURSE TITLE: Algebra II

TITLE OF SOFTWARE PACKAGE NEEDED FOR THE LESSON:
Graphing Equations by Conduit (or any graphing utility which allows entry of linear equations in alternate forms).

TYPE OF ACTIVITY: Classroom/Lab

SUGGESTED TIMELINE: 1-2 periods

PREREQUISITE SKILLS:
The student should be:

- familiar with the vocabulary of linear functions: slope, x and y intercepts, point, quadrants, horizontal, vertical
- able to graph linear functions
- able to recognize and interpret linear equations presented in both slope-intercept and point-slope forms
- familiar with the 'Equation Plotter' portion of Graphing Equations (or a comparable graphing utility)

PIMMS

REVIEWING LINEAR EQUATIONS
LESSON 3

LABORATORY EXERCISES

Use the 'Equation Plotter' option of Graphing Equations to check your graph.

1. Write an equation for the line and graph.

a) m = 1/2
point(0,2)

b) point(3,-1)
m = 2

c) m = 0
b = -3

d) m undefined
point(-1,0)

e) point(-1,-2)
point(-3,-1)

f) m = -1/3
point(-2,1)

g) point(3,2)
point(0,1)

h) m = 2
b = -3

i) point(3,2)
point(-6,2)

j) point(-3,1)
point(-3,-4)

2. Give your response. Then use the Equation Plotter to check.

a) The graph of a line with slope 2 containing (1,-1) is parallel to y = 2x. True or False

b) The graph of a line with m = -1/4 and b = -2 contains (-8,4). True or False

c) The graph of y-3 = 2(x+1) is _____ to the graph of the line containing (0,1) with slope -1/2.

d) The graph of y = -x is parallel to the graph of y-0 = x+0. True or False

e) Give the relationship of the graphs of line 1 and line 2.

line 1: m = 2, contains (-1,-2)
line 2: y = 2x + 3

f) Give the relationship of the graphs of line 1 having slope 2 and containing (2,3) to line 2 having equation y-6 = (-1/2)x.

g) The graphs of line 1 and line 2 contain a common point. Give the coordinates of this point.

line 1: y-3 = -1/3(x+3)
line 2: y = (2/3)x-1

h) The graph of y+2 = 1/4(x+4) contains (0,-1). True or False

ANSWER KEY

REVIEWING LINEAR EQUATIONS
LESSON 3

LABORATORY EXERCISES

Use the 'Equation Plotter' option of Graphing Equations to check your graph.

1. Write an equation for the line and graph. *See next page*

a) m = 1/2
point(0,2)

b) point(3,-1)
m = 2

c) m = 0
b = -3

d) m undefined
point(-1,0)

e) point(-1,-2)
point(-3,-1)

f) m = -1/3
point(-2,1)

g) point(3,2)
point(0,1)

h) m = 2
b = -3

i) point(3,2)
point(-6,2)

j) point(-3,1)
point(-3,-4)

2. Give your response. Then use the Equation Plotter to check.

a) The graph of a line with slope 2 containing (1,-1) is parallel to y = 2x. (True) or False

b) The graph of a line with m = -1/4 and b = -2 contains (-8,4). True or (False)

c) The graph of y-3 = 2(x+1) is *perpendicular* to the graph of the line containing (0,1) with slope -1/2.

d) The graph of y = -x is parallel to the graph of y-0 = x+0. True or (False)

e) Give the relationship of the graphs of line 1 and line 2.

line 1: m = 2, contains (-1,-2)
line 2: y = 2x + 3

The lines are parallel.

f) Give the relationship of the graphs of line 1 having slope 2 and containing (2,3) to line 2 having equation y-6 = (-1/2)x.

The lines are perpendicular.

g) The graphs of line 1 and line 2 contain a common point. Give the coordinates of this point.

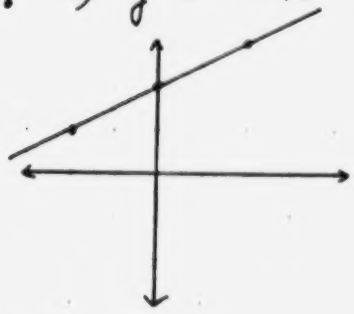
line 1: y-3 = -1/3(x+3)
line 2: y = (2/3)x-1

Common point is (3,1).

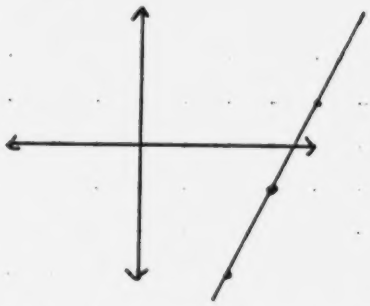
h) The graph of y+2 = 1/4(x+4) contains (0,-1). (True) or False

ANSWER KEY
LABORATORY EXERCISES

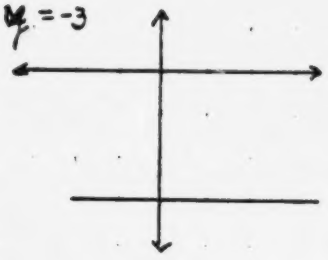
1. a) $y - 2 = \frac{1}{2}x$



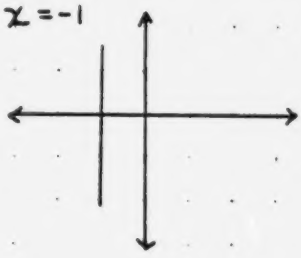
b) $y + 1 = 2(x - 3)$



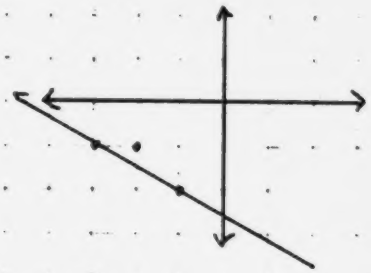
c) $y = -3$



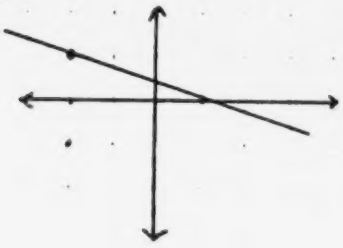
d) $x = -1$



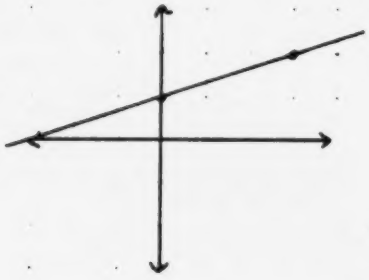
e) $y + 2 = -\frac{1}{2}(x + 1)$
or
$y + 1 = -\frac{1}{2}(x + 3)$



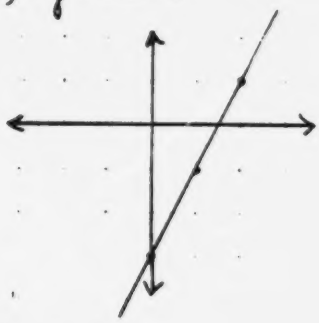
f) $y - 1 = -\frac{1}{3}(x + 2)$



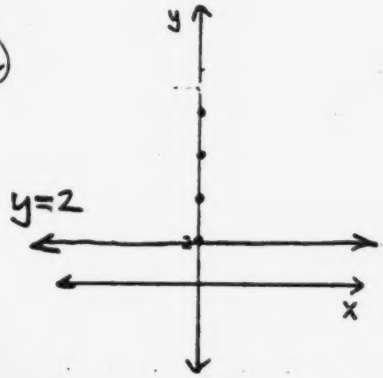
g) $y - 2 = \frac{1}{3}(x - 3)$
or
$y - 1 = \frac{1}{3}x$



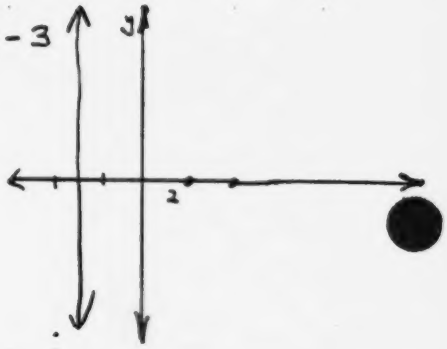
h) $y = 2x - 3$



i)



j) $x = -3$



REVIEWING LINEAR EQUATIONS
HOMEWORK EXERCISES
LESSON 3

1. Write an equation for the line and draw its graph.

a) m = 1/3, b = 0

b) point = (0,5), m = 0

c) m = undefined, point = (0,0)

d) point = (4,-1), point = (3,-2)

e) m = -1, b = -1

f) m = 0, b = -3

g) point = (0,0), point = (-2,2)

h) m = -3, point = (2,3)

i) point = (1,-2), point = (-1,2)

2.

a) Describe the graph of a line whose slope is undefined.

b) For the slope to be undefined, what would be true in the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$?

c) Write an equation for the line that has slope 1 and contains (0,6). Graph.

d) Describe the graph of a line whose slope is found using $m = \frac{3-3}{-3-3}$.

Write an equation for the line which has the same slope but which also contains the point (0,5). Graph.

e) Write an equation for the line containing (5,-1) and (3,2). Graph.

f) Write an equation for a line with y-intercept (0,3) whose graph is parallel to the graph of y = -3x-2.

g) The y-intercept of the graph of y = 4x+2 is (0,2). The x-intercept is _____. How can you use the linear equation (rather than the graph itself) to find the x-intercept?

h) Given y-intercept (0,-1) and x-intercept (2,0), write an equation for the line. Graph.

ANSWER KEY

REVIEWING LINEAR EQUATIONS
HOMEWORK EXERCISES
LESSON 3

1. Write an equation for the line and graph. (See next page)

a) $m = 1/3$, $b = 0$

b) point = (0,5), $m = 0$

c) m = undefined, point = (0,0)

d) point = (4,-1), point = (3,-2)

e) $m = -1$, $b = -1$

f) $m = 0$, $b = -3$

g) point = (0,0), point = (-2,2)

h) $m = -3$, point = (2,3)

i) point = (1,-2), point = (-1,2)

2.

a) Describe the graph of a line whose slope is undefined.

The graph is parallel to the y-axis.

b) For the slope to be undefined, what would be true in the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$?

$x_2 = x_1$, $y_2 \neq y_1$

c) Write an equation for the line that has slope 1 and contains (0,6). Graph.

$y = x + 6$

d) Describe the graph of a line whose slope is found using $m = \frac{3-3}{-3-3}$.

Since $y_2 = y_1$ and $x_2 \neq x_1$, the line is parallel to the x-axis.

Write an equation for the line which has the same slope but which also contains the point (0,5). Graph. $y = 5$

e) Write an equation for the line containing (5,-1) and (3,2). Graph.

$y + 1 = -\frac{3}{2}(x - 5)$ or, $y - 2 = -\frac{3}{2}(x - 3)$

f) Write an equation for a line with y-intercept (0,3) whose graph is parallel to the graph of $y = -3x - 2$. $y = -3x + 3$, among others

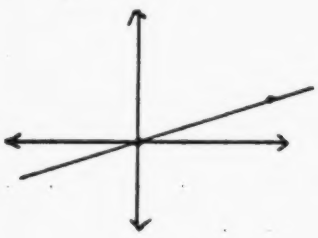
g) The y-intercept of the graph of $y = 4x + 2$ is (0,2). The x-intercept is $(-\frac{1}{2}, 0)$. How can you use the linear equation (rather than the graph itself) to find the x-intercept? Use $x = -\frac{b}{m}$.

h) Given y-intercept (0,-1) and x-intercept (2,0), write an equation for the line. Graph. $y = \frac{1}{2}x - 1$

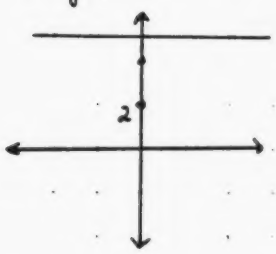
ANSWER KEY
HOMEWORK EXERCISES

1.

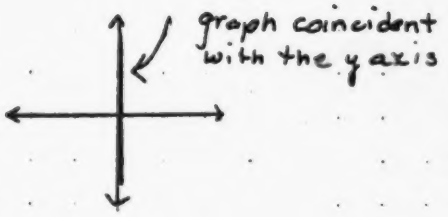
a) $y = \frac{1}{3}x$



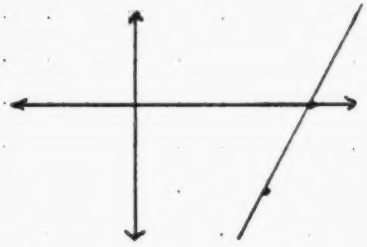
b) $y = 5$



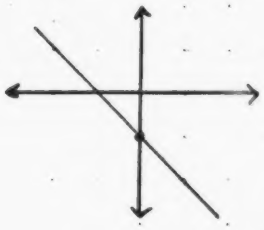
c) $x = 0$



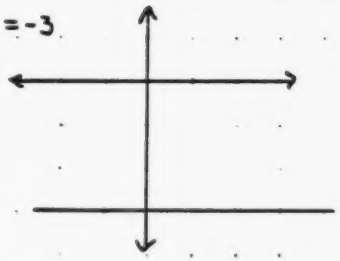
d) $y+1 = 2(x-4)$
or $y+2 = 2(x-3)$



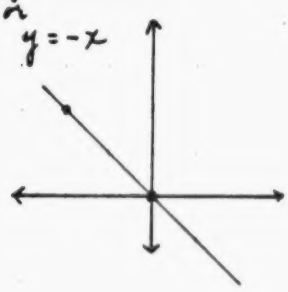
e) $y = -x - 1$



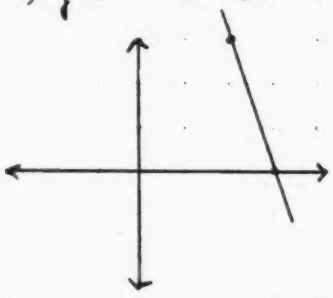
f) $y = -3$



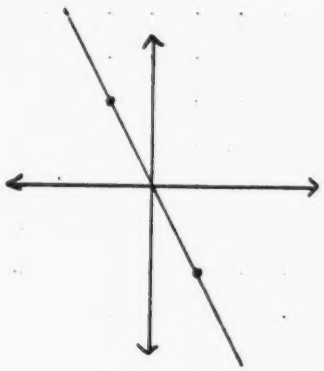
g) $y - 2 = -1(x+2)$
or $y = -x$



h) $y - 3 = -3(x+2)$



i) $y+2 = -2(x-1)$
or $y-2 = -2(x+1)$



REVIEWING LINEAR EQUATIONS
LESSON 4

OBJECTIVE: To review the relationship between points in a line, its graph, and its equation.

To complete the three levels of proficiency in the 'Lines' option of the 'Linear and Quadratic Graphs' section of Graphing Equations.

To review linear equations via competitive team play of the Green Globs option in Graphing Equations.

COURSE TITLE: Algebra II

TITLE OF SOFTWARE PACKAGE NEEDED FOR THE LESSON:
Graphing Equations (Conduit)

TYPE OF ACTIVITY: Classroom/Lab

SUGGESTED TIMELINE: 1 period

PREREQUISITE SKILLS:
The student should be:

- familiar with the terminology of linear functions: slope, x- and y-intercepts, point, quadrants, horizontal, vertical
- able to graph linear functions
- able to recognize and interpret linear equations presented in both slope-intercept and point-slope forms
- familiar with the 'Linear and Quadratic Graphs' portion of Graphing Equations.
- familiar with the 'Green Globs' option in Graphing Equations

PIMMS

REVIEWING LINEAR EQUATIONS
LESSON 4
COMPUTER EXERCISES

1. In groups of 2 or 3, use the 'Lines' option of the 'Linear and Quadratic Graphs' section of Graphing Equations to practice writing linear equations in both slope-intercept and point-slope forms. One team member should act as the recorder. The correct equations used, as well as any incorrect attempts, are to be recorded for each of the twenty levels of skill found within the three general levels of proficiency.

   Complete as many sections as possible and submit your team results to the game manager.

2. In groups of 2 or 3, use the 'Green Globs' option of Graphing Equations. The objective here is to achieve as high a score as possible by writing linear equations that 'shoot' the greatest number of globs. Record the equations, the game level (Novice or Expert), and the team score and give them to the game manager.

TITLE: DISCOVERY LEARNING IN TRIGONOMETRY
PUBLISHER: CONDUIT
ADDRESS: THE UNIVERSITY OF IOWA
OAKDALE CAMPUS
ROOM 6700
IOWA CITY, IA 52244
PRICE: $75.00
COMPUTER: APPLE II Plus/e/C (IBM PC, PC XT, PCjr version also available)
PIMMS RATING: EXCELLENT

PROGRAM APPROPRIATE FOR:

ABILITY GROUP	: LOW, MIDDLE, HIGH
COURSE	: ADVANCED ALGEBRA, TRIGONOMETRY, ANALYTIC GEOMETRY, CALCULUS
FORMAT	: MAINLY DISCOVERY/LAB, BUT MAY BE USED FOR FOR CLASS DEMO/INSTRUCTIONAL
CHARACTERISTICS	: COMES WITH A BACKUP DISKETTE, EASY TO USE EVEN THOUGH DIRECTIONS ARE NOT PROVIDED, SUPPORT MATERIAL COMES WITH THE PROGRAM, HAS A GRAPHING UTILITY, HAS A DEMO MODE, DISK DEPENDENT

ADDITIONAL PERIPHERALS NEEDED: COLOR MONITOR PREFERRED

This program was designed as a laboratory activity for individuals but may be used successfully for group instruction. It encourages discovery learning by generating a number of examples in a short time so that students can recognize patterns. The various graphical displays including a fill-in option are useful in a calculus class for discussing the area between two curves. Laboratory lessons are provided but are not detailed.

Topics covered in the exercises include discovery activities with trigonometric functions and their related amplitudes, periods, phase shifts, and center lines. The program helps the student to establish relationships such as reciprocal trigonometric relationships and trig identities. The program also contains a unit on polar equations and their graphs in the same discovery style.

A convenient feature of the program is a STOP / CONTINUE GRAPHING command that permits the teacher to ask students to quickly sketch a graph before actually showing the graph. The TRIG PROBLEM section provides an excellent exercise in reverse thinking. It displays a trig graph and asks the student to formulate an appropriate equation for the graph.

This program should be included in any core collection of mathematics software.

DISCOVERY LEARNING IN TRIGONOMETRY
DIRECTIONS FOR USE

Insert the diskette in the disk drive and turn on the computer. Once the credits are on the screen, hit Return. A listing of all the available programs on the diskette, called the Menu, will appear. Enter the number of the program you wish to see and press Return. Up to this point the program diskette must be in the disk drive. The space bar is used to scroll through the program.

PROGRAM 1 TRIG DEMO
Trig Demo illustrates the graphs of various trigonometric functions using the various graphing modes available in the package. The program continuously displays functions until the space bar is pressed. This is a good program for introducing a unit on the graphs of the trigonometric functions so that students can see where they are going and what to expect.

PROGRAM 2 GENERAL TRIG
General Trig is a user-directed graphing utility. All functions must be entered using BASIC notation. This package is not as versatile as others since there are only three options given for boundaries-

Small: x between 0 and 6.28 y between -3 and 3;
Medium: x between -6.28 and 6.28 y between -4 and 4;
Large: x between -15 and 15 y between -10 and 10.

The three graphing options available in Trig Demo-

Dots: uses dots to draw the graph (the quickest mode);
Lines: fills in between points to give a smoother picture;
Full: shades in under the graph.

Available color options are listed for the background, the axes, and the graph. Several curves may be graphed on the same set of axes. The fill-in option can be used to introduce the concept of area under the curve or to contrast families of curves.

PROGRAM 3 TRIG TREND
Trig Trend graphs a family of three trigonometric functions. The function entered must contain a constant c such as y = c*cos(c*x). The program automatically assigns c = 1, c = 2, and c = 3, then sketches the three graphs in that family of curves. You may not enter your own values for c but may go back to the General Trig program to enter other values for c in the same family. This is a good demonstration program to use before you begin to study the effects of parameters on the graph of a trig function.

PROGRAM 4 TRIG PROBLEM SOLVING
Trig Problem Solving randomly selects a function and plots it on the screen. The student then must identify the equation of the graph. Once the student types in a response the computer graphs the guess and tells the student whether or not the guess was correct. This is an excellent closing exercise to the graphing unit. Levels of problem solving range from easy sine and cosine graphs to all six trigonometric functions with a phase shift. Another option really challenges the student (and the teacher!) by moving a point through the locus without tracing the path of the

point. The space bar can be used to stop a graph before completion which increases the opportunity for more challenge. In this section there does not appear to be any way to have the computer provide the answer. The student must guess, reestimate, and reenter the revised function until a correct response is given.

PROGRAM 5 POLAR DEMO
Polar Demo, like Trig Demo, continuously demonstrates the capabilities of the polar graphing programs. Students seem to be fascinated by the various types of polar graphs displayed. Less familiar polar functions such as the star shape are included in this slide show. This program can be run as a class demo to introduce a unit on polar functions so that students can appreciate the complexity and beauty of the polar curves. The program also can be run after the polar unit has been completed with students being challenged to name the equation represented by the graph being sketched. (sketching of the curves does take a bit of time.)

PROGRAM 6 GENERAL POLAR
General Polar plots polar equations of the form R(A)= sin (2*A). Unfortunately, this program does not graph equations in which R is squared. (The MECC program, Mathematics Volume I, Polar, does accept polar equations where R is squared and may be used along with this program to fill in that gap.) A maximum radius must be selected from 1 to 99 and determines how large your graph will appear on the screen. The user must analyze the function to be entered and determine whether other graphs will be plotted on the same set of axes before selecting a radius. Once the various options are chosen for the first graph, they may not be changed for subsequent graphs. For example, to graph y = sin(2*A), y = 2*sin(2*A), and y = -5*sin(2*A), select a radius of 5, not 1.

Optional features such as background color, grid color, and graph color also may be selected. The graph may be drawn with or without degree markers. The number of angle rotations spanned by A may be varied and may include fractional parts of a rotation.

PROGRAM 7 POLAR TREND
Polar Trend, like Trig Trend, plots families of polar equations using the parameter c, where c = 1, 2, and 3. Since c is restricted in value, this program is useful mainly for class demos.

To return to the Main Menu the program diskette must be in the disk drive. The Quit option exits the program entirely.

EXPLORING LIMACONS

OBJECTIVES: To explore the nature of the graphs of the polar equations:

$R(A) = a + b * \cos(A)$

$R(A) = a + b * \sin(A)$

for the three cases when

$ABS(a) = ABS(b)$, $ABS(a) > ABS(b)$, $ABS(a) < ABS(b)$.

COURSE TITLE: Precalculus or Analytic Geometry

TITLE OF SOFTWARE PACKAGE NEEDED FOR THE LESSON:
Discovery Learning in Trigonometry (Conduit)
or any polar graphing utility.

TYPE OF ACTIVITY: Discovery/Lab
Students should be grouped two or three to a computer.

SUGGESTED TIME: 1 Class Period

PREREQUISITE SKILLS:
The student should:

- understand the concept of the polar plane for locating points
- be able to plot polar coordinates of the form (r,A) for $r > 0$ and for $r < 0$.
- know the meaning of the sine and cosine functions
- know the definition of symmetry
- be able to enter the polar equations using BASIC notation

NOTE TO THE TEACHER:
Once the laboratory work has been completed the teacher should classify the curves as limacons and should describe the shapes of the curves when $|a| = b$ (cardioids), when $|a| > |b|$ (graph surrounds the origin like a stretched out circle), when $|a| < |b|$ (limacon with a loop).

EXPLORING LIMACONS

Each of the functions used in these exercises is in the form

$$R(A) = a + b * COS(A)$$
or $$R(A) = a + b * SIN(A)$$

GROUP I:
a) $R(A) = 2 + 2 * COS(A)$
b) $R(A) = 3 - 3 * SIN(A)$

Group II:
c) $R(A) = 5 + 3 * COS(A)$
d) $R(A) = -4 + 2 * SIN(A)$

GROUP III:
e) $R(A) = -1 - 2 * COS(A)$
f) $R(A) = 2 + 4 * SIN(A)$

1) Identify the values of a and of b for each of the six functions listed above.

	a	b
a)		
b)		
c)		
d)		
e)		
f)		

2) What appears to be the rule used to classify the equations into Groups I, II, or III?

3) Load the "General Polar" program of Discovery Learning in Trigonometry (or any other polar graphing utility). Enter each of the functions listed above into the computer and record the following for graphs a, b, c, d, e, and f:

Graph	a	b	c	d	e	f
the point where the plot of the graph begins						
the line of symmetry of the graph						
the orientation of the graph (predominantly to the right or left of the 90-degree line; predominantly above or below the 0-degree line)						
the shape of the curve (record a sketch, including intercepts)						
the intercepts of the curve (at the 0, 90, 180, 270 degree lines)						
the intercepts of the curve at any other line						

Discuss the following questions with your group or your partner and write down your agreed upon answer.
Be prepared to support your responses to the rest of the class.

4) Review your answer to question 2.
Do you want to revise it in any way? Why or why not?

5) Determine how the signs of a and b (called parameters) affect the starting point of the graph.

6) For what function, sine or cosine, does the graph appear to have symmetry to the 90-degree line?

7) For what function, sine or cosine, does the graph appear to have symmetry to the 0-degree line?

8) Determine how the signs of a and b affect the orientation of the graphs.

9) How many intercepts are there when:
the absolute value of a equals the absolute value of b?
the absolute value of a is greater than the absolute value of b?
the absolute value of a is less than the absolute value of b?

10) Make a conjecture about how the intercepts are related to the values of a and b.

11) Test your conjecture by applying it to the equations R(A) = -2 + 2 * SIN(A) and R(A) = 3 - 4 * COS(A).
Enter and record their graphs.

12) Despite the orientation of the graph, three basic shapes appeared. Describe the three different shapes of the graphs.

13) Determine the relationship between the absolute value of a and the absolute value of b for each of the three shapes.

14) What is the difference, if any, between the graphs of R(A) = a + ABS(b)*COS(A) and R(A) = -a + ABS(b)*COS(a)? (Compare the graph of R(A) = 3 - 4 * COS(A) sketched previously to R(A) = -3 - 4 * COS(A).) Does the same relationship hold true for limacon equations containing the sine function?

15) Create an equation in the form R(A) = a + b * cos(A) or R(A) = a + b * sin(A) which satisfies the following criteria. Test your equation by entering it into the computer and plotting its graph. Does it meet all the criteria? If not, refine your guess and check again. Make sure each group member can justify your response to the rest of the class.

a) predominantly to the right of the 90-degree line
intercepts: (0, 90°) (2, 0°) (1, 90°) (1, 270°)

b) 90-degree line is the line of symmetry
intercepts: (0, 0°) (1, 0°) (-1, 90°) (1, 180°) (3, 270°)

EXPLORING LIMACONS

Each of the functions used in these exercises is in the form

$$R(A) = a + b * COS(A)$$
or $$R(A) = a + b * SIN(A)$$

GROUP I:
a) $R(A) = 2 + 2 * COS(A)$
b) $R(A) = 3 - 3 * SIN(A)$

Group II:
c) $R(A) = 5 + 3 * COS(A)$
d) $R(A) = -4 + 2 * SIN(A)$

GROUP III:
e) $R(A) = -1 - 2 * COS(A)$
f) $R(A) = 2 + 4 * SIN(A)$

1) Identify the values of a and of b for each of the six functions listed above.

	a	b
a)	$a=2$	$b=2$
b)	$a=3$	$b=-3$
c)	$a=5$	$b=3$
d)	$a=-4$	$b=2$
e)	$a=-1$	$b=-2$
f)	$a=2$	$b=4$

2) What appears to be the rule used to classify the equations into Groups I, II, or III?

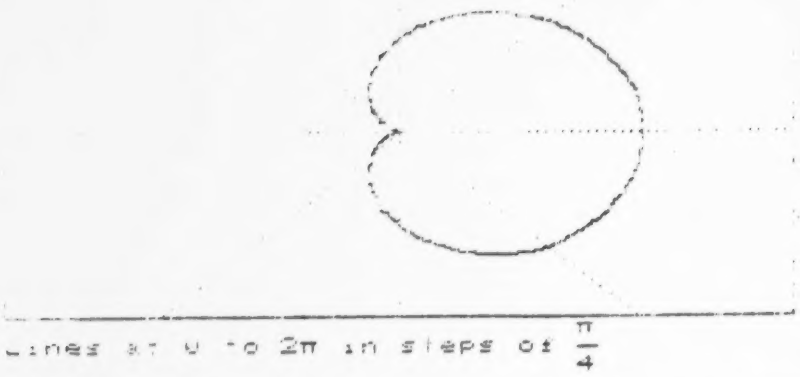
GROUP I: $|a|=|b|$
GROUP II: $|a|>|b|$
GROUP III: $|a|<|b|$

3) Load the "General Polar" program of Discovery Learning in Trigonometry (or any other polar graphing utility). Enter each of the functions listed above into the computer and record the following for graphs a, b, c, d, e, and f:

Graph	a	b	c	d	e	f
the point where the plot of the graph begins	$(4,0^\circ)$	$(3,0^\circ)$	$(8,0^\circ)$	$(-4,0^\circ)$	$(-3,0^\circ)$	$(2,0^\circ)$
the line of symmetry of the graph	0° LINE	90°-LINE	0°-LINE	90°-LINE	0°-LINE	90°-LINE
the orientation of the graph (predominantly to the right or left of the 90-degree line; predominantly above or below the 0-degree line)	PREDOMINANTLY: RT. OF 90° LINE	BELOW 0° LINE	RT. OF 90° LINE	ABOVE 0° LINE	LEFT OF 90° LINE	ABOVE 0° LINE
the shape of the curve (record a sketch, including intercepts)						
the intercepts of the curve (at the 0, 90, 180, 270 degree lines)	$(4,0^\circ)$ $(2,90^\circ)$ $(0,180^\circ)$ $(2,270^\circ)$	$(3,0^\circ)$ $(0,90^\circ)$ $(-3,180^\circ)$ $(6,270^\circ)$	$(8,0^\circ)$ $(5,90^\circ)$ $(2,180^\circ)$ $(5,270^\circ)$	$(-4,0^\circ)$ $(-2,90^\circ)$ $(-4,180^\circ)$ $(-6,270^\circ)$	$(-3,0^\circ)$ $(-1,90^\circ)$ $(1,180^\circ)$ $(-1,270^\circ)$	$(2,0^\circ)$ $(6,90^\circ)$ $(2,180^\circ)$ $(-2,270^\circ)$
the intercepts of the curve at any other line	NONE	NONE	NONE	NONE	$(0,120^\circ)$ $(0,240^\circ)$ SAME AS $(0,0^\circ)$	$(0,210^\circ)$ $(0,330^\circ)$

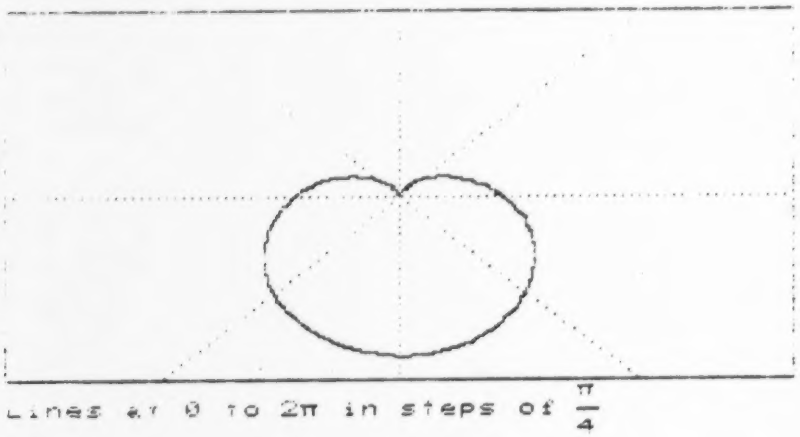
3. a)

$R(A) = 2 + 2\cos(A)$



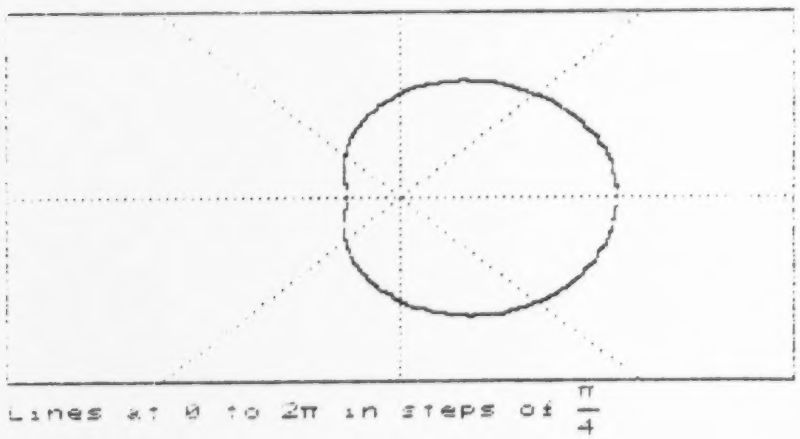
b)

$R(A) = 3 - 3\sin(A)$



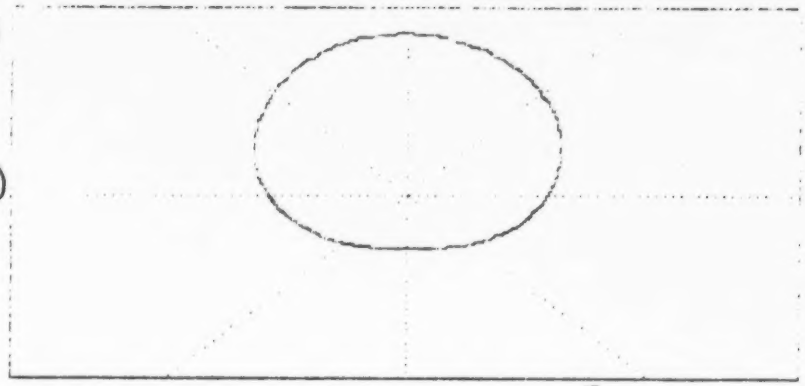
c)

$R(A) = 5 + 3\cos(A)$



d)

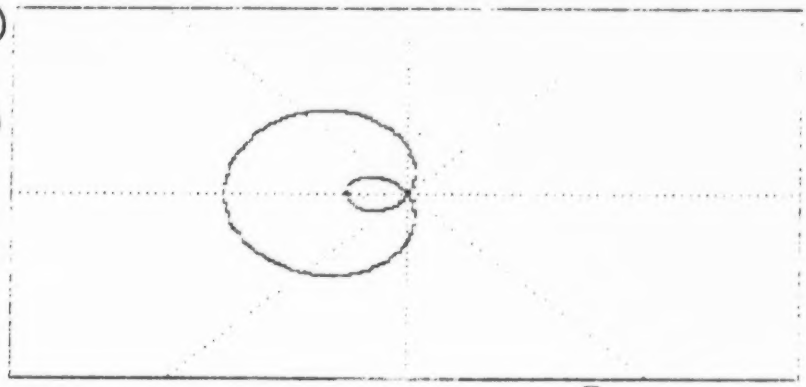
$R(A) = -4 + 2\,SIN(A)$



Lines at 0 to 2π in steps of $\frac{\pi}{4}$

e)

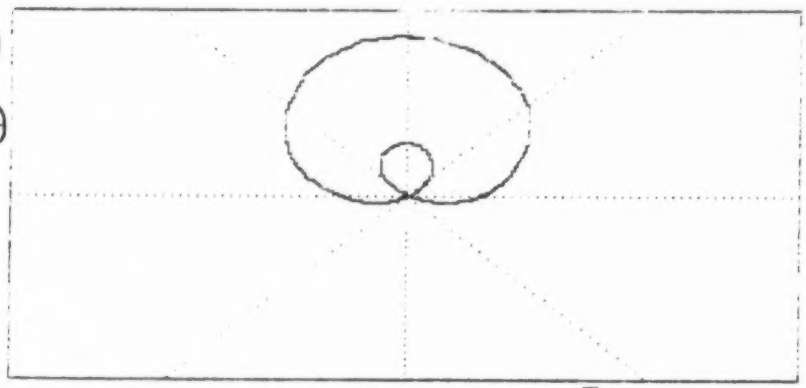
$R(A) = -1 - 2\,COS(A)$



Lines at 0 to 2π in steps of $\frac{\pi}{4}$

f)

$R(A) = 2 + .4\,SIN(A)$



Lines at 0 to 2π in steps of $\frac{\pi}{4}$

Discuss the following questions with your group or your partner and write down your agreed upon answer.
Be prepared to support your responses to the rest of the class.

4) Review your answer to question 2.
Do you want to revise it in any way? Why or why not?

5) Determine how the signs of a and b (called parameters) affect the starting point of the graph.
FOR $R(A) = a + b\cos(A)$ THE STARTING POINT IS AT $(a+b, 0^\circ)$
FOR $R(A) = a + b\sin(A)$ THE STARTING POINT IS AT $(a, 0^\circ)$

6) For what function, sine or cosine, does the graph appear to have symmetry to the 90-degree line?
SINE

7) For what function, sine or cosine, does the graph appear to have symmetry to the 0-degree line?
COSINE

8) Determine how the signs of a and b affect the orientation of the graphs. a HAS NO EFFECT

FOR $b>0$: PREDOMINANTLY ABOVE THE 0° LINE FOR SINE
TO RIGHT OF 90° LINE FOR COSINE

FOR $b<0$: PREDOMINANTLY BELOW THE 0° LINE FOR SINE
TO LEFT OF THE 90° LINE FOR COSINE

9) How many intercepts are there when:
the absolute value of a equals the absolute value of b? 4
the absolute value of a is greater than the absolute value of b? 4
the absolute value of a is less than the absolute value of b? 5

10) Make a conjecture about how the intercepts are related to the values of a and b.
FOR COSINE: $(a+b, 0^\circ)$ $(a, 90^\circ)$ $(a-b, 180^\circ)$ $(a, 270^\circ)$
FOR SINE: $(a, 0^\circ)$ $(a+b, 90^\circ)$ $(a, 180^\circ)$ $(a-b, 270^\circ)$

11) Test your conjecture by applying it to the equations
R(A) = -2 + 2 * SIN(A) and R(A) = 3 - 4 * COS(A).
Enter and record their graphs.

12) Despite the orientation of the graph, three basic shapes appeared. Describe the three different shapes of the graphs.

WHEN $|a|=|b|$ HEART-SHAPED LIMACON (CARDIOID)

WHEN $|a|>|b|$ SURROUNDS THE ORIGIN LIKE A STRETCHED-OUT CIRCLE

WHEN $|a|<|b|$ LIMACON WITH A LOOP

13) Determine the relationship between the absolute value of a and the absolute value of b for each of the three shapes.

SEE ABOVE

14) What is the difference, if any, between the graphs of R(A) = a + ABS(b)*COS(A) and R(A) = -a + ABS(b)*COS(a)? (Compare the graph of R(A) = 3 - 4 * COS(A) sketched previously to R(A) = -3 - 4 * COS(A).) Does the same relationship hold true for limacon equations containing the sine function?

THEY ARE THE SAME

15) Create an equation in the form R(A) = a + b * cos(A) or R(A) = a + b * sin(A) which satisfies the following criteria. Test your equation by entering it into the computer and plotting its graph. Does it meet all the criteria? If not, refine your guess and check again. Make sure each group member can justify your response to the rest of the class.

a) predominantly to the right of the 90-degree line
intercepts: (2, 0°) (1, 90°) (0, 180°) (1, 270°)

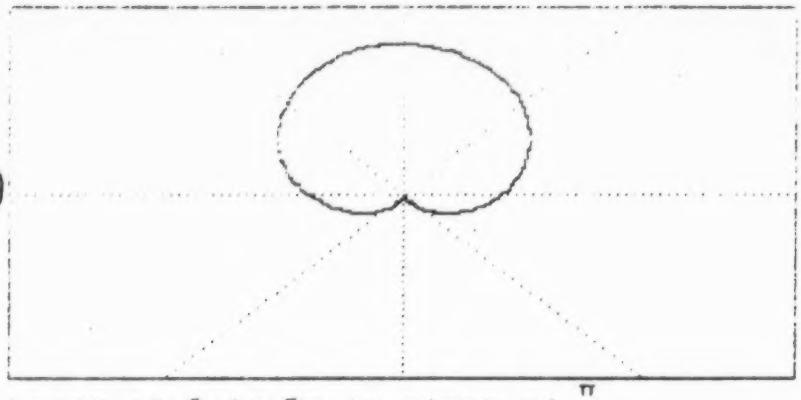
$R(A) = 1 + \cos(A)$

b) 90-degree line is the line of symmetry
intercepts: (0, 0°) (1, 0°) (-1, 90°) (1, 180°) (3, 270°)

$R(A) = 1 - 2\sin(A)$

(11)

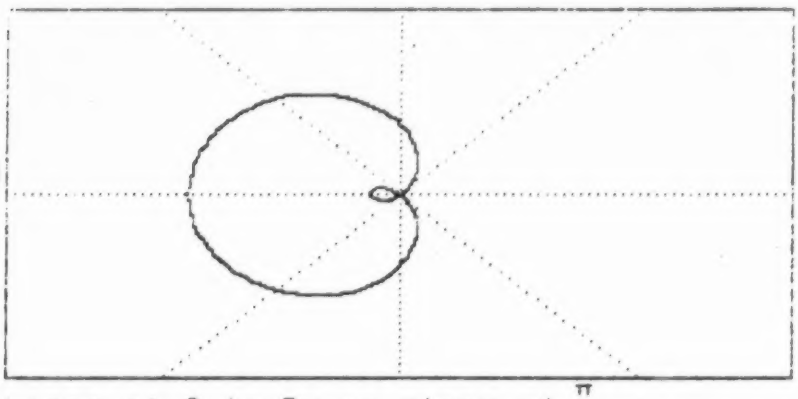
$R(A) = -2 + 2\sin(A)$



Lines at 0 to 2π in steps of $\frac{\pi}{4}$

(11) & (14)

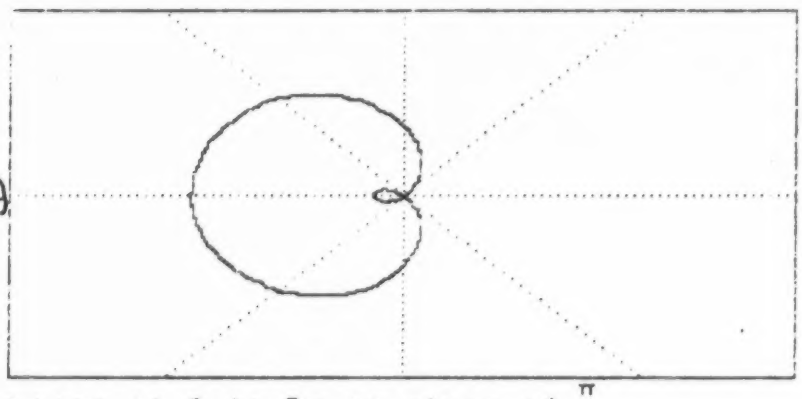
$R(A) = 3 - 4\cos(A)$



Lines at 0 to 2π in steps of $\frac{\pi}{4}$

(14)

$R(A) = -3 - 4\cos(A)$



Lines at 0 to 2π in steps of $\frac{\pi}{4}$

EXPLORING LIMACONS
HOMEWORK EXERCISES

For each of the following equations of limacons:

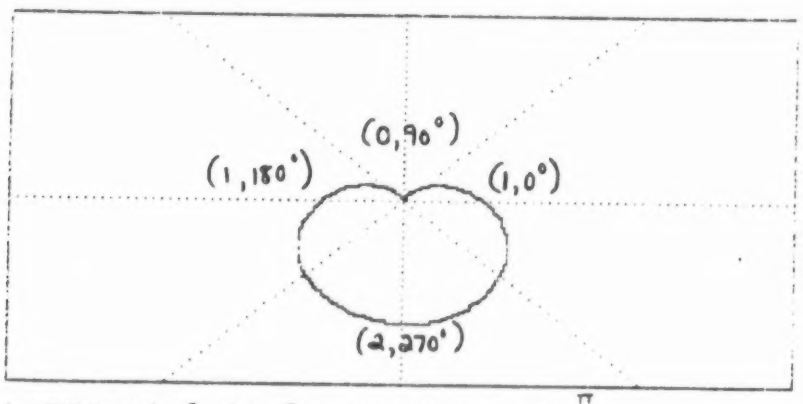
a) determine its axis of symmetry
b) determine the value of r when A = 0 degrees (the starting point of the graph)
c) determine the shape of the graph (cardioid, limacon with a loop, limacon which surrounds the pole)
d) determine the orientation of the graph (predominately to the right or to the left of the 90-degree line; predominately above or below the 90-degree line
e) give the intercepts
f) SKETCH the curve on polar paper using the above information. (You may experiment with plotting a few points if necessary. Use a calculator to aid computation.)

1) R(A) = 3 + 2 * COS(A)
2) R(A) = 4 - 4 * SIN(A)
3) R(A) = -2 - 4 * SIN(A)
4) R(A) = 1 + 2 * COS(A)
5) R(A) = -3 + 3 * COS(A)
6) R(A) = 5 + 3 * SIN(A)

7) <u>Challenge Question</u>
How would the graphs of 1)- 6) above change if R(A) were squared in each equation? Be specific and cite examples to support your answer.

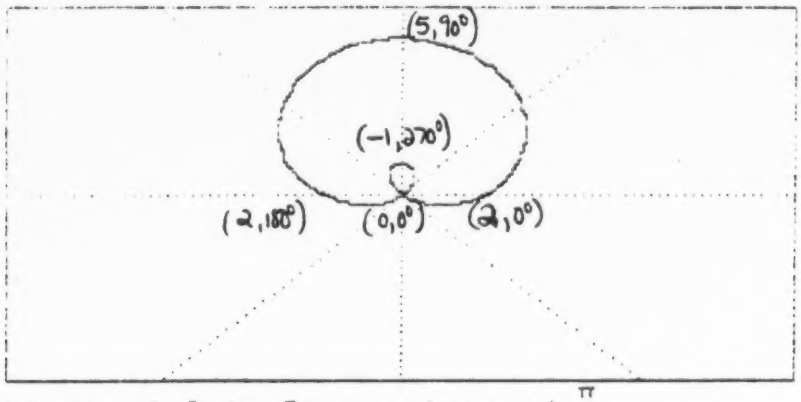
8). Determine an equation for each of the following polar graphs.

a)



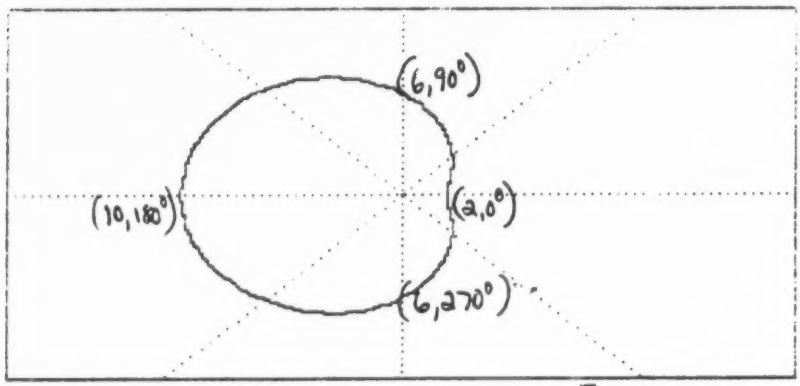
Lines at 0 to 2π in steps of $\frac{\pi}{4}$

b)



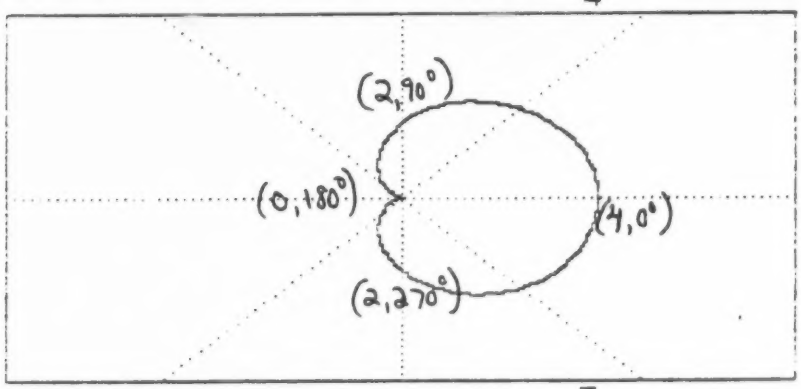
Lines at 0 to 2π in steps of $\frac{\pi}{4}$

c)



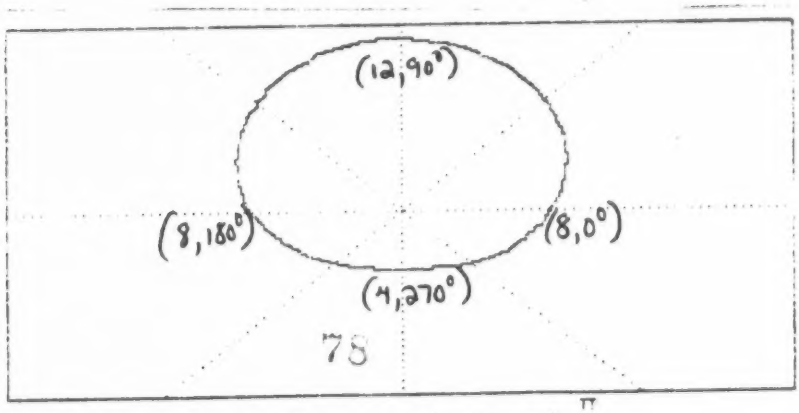
Lines at 0 to 2π in steps of $\frac{\pi}{4}$

d).



Lines at 0 to 2π in steps of $\frac{\pi}{4}$

e)



Lines at 0 to 2π in steps of $\frac{\pi}{4}$

ANSWER KEY

EXPLORING LIMACONS
HOMEWORK EXERCISES

For each of the following equations of limacons:

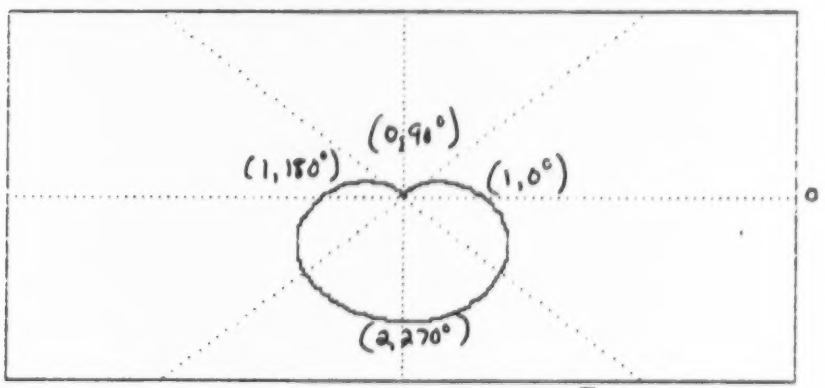
a) determine its axis of symmetry
b) determine the value of r when A = 0 degrees (the starting point of the graph)
c) determine the shape of the graph (cardioid, limacon with a loop, limacon which surrounds the pole)
d) determine the orientation of the graph (predominately to the right or to the left of the 90-degree line; predominately above or below the 90-degree line
e) give the intercepts
f) SKETCH the curve on polar paper using the above information. (You may experiment with plotting a few points if necessary. Use a calculator to aid computation.)

1) R(A) = 3 + 2 * COS(A)
2) R(A) = 4 - 4 * SIN(A)
3) R(A) = -2 - 4 * SIN(A) See p.74
4) R(A) = 1 + 2 * COS(A)
5) R(A) = -3 + 3 * COS(A)
6) R(A) = 5 + 3 * SIN(A)

7) <u>Challenge Question</u>
How would the graphs of 1)- 6) above change if R(A) were squared in each equation? Be specific and cite examples to support your answer. See p.76

8). Determine an equation for each of the following polar graphs.

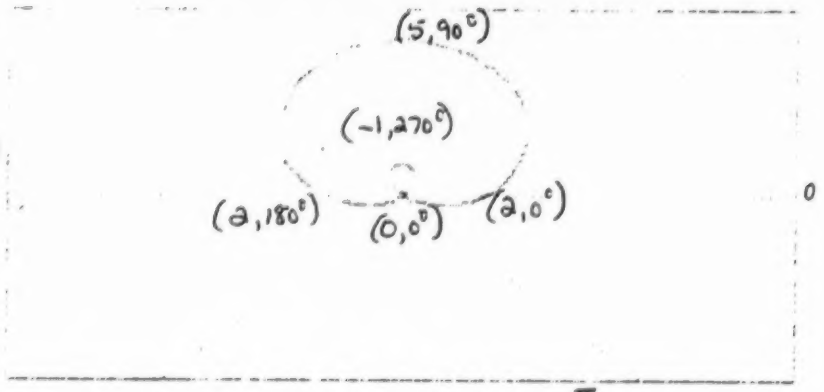
a)
R(A)= 1-SIN(A)



Lines at 0 to 2π in steps of $\frac{\pi}{4}$

b)

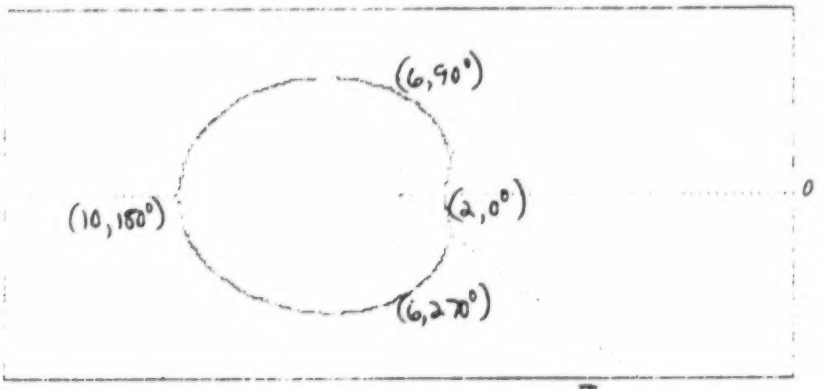
$R(A) = 2 + 3\sin(A)$



Lines at Ø to 2π in steps of $\frac{\pi}{4}$

c)

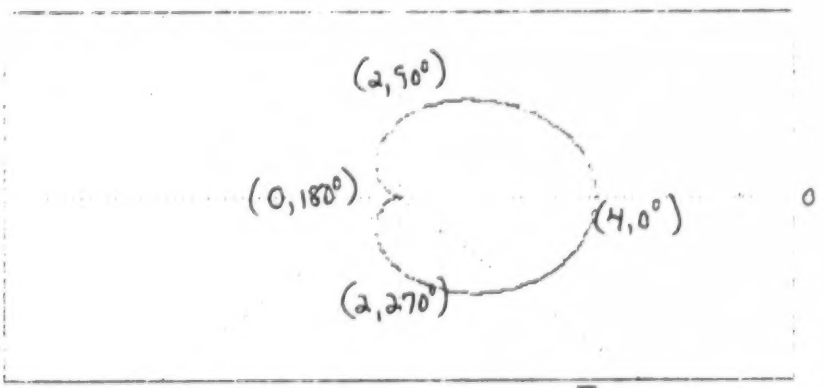
$R(A) = 6 - 4\cos(A)$



Lines at Ø to 2π in steps of $\frac{\pi}{4}$

d)

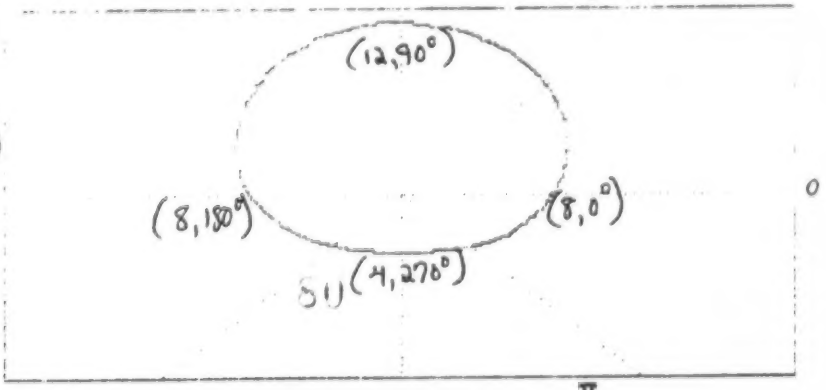
$R(A) = 2 + 2\cos(A)$



Lines at Ø to 2π in steps of $\frac{\pi}{4}$

e)

$R(A) = 8 + 4\sin(A)$

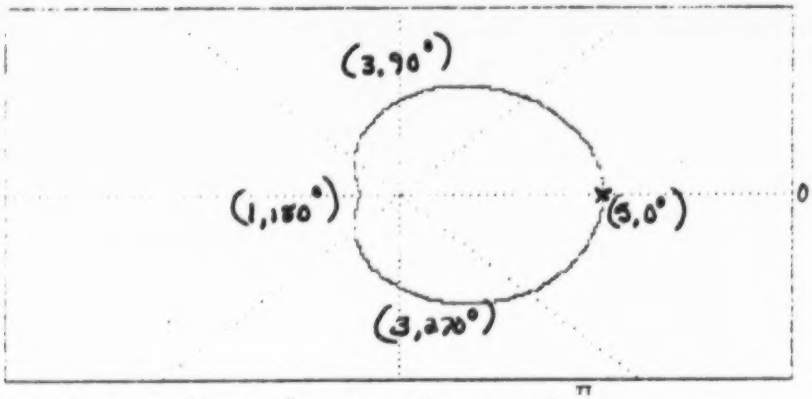


Lines at Ø to 2π in steps of $\frac{\pi}{4}$

ANSWER KEY - HOMEWORK EXERCISES 1-7

1)

$R(A) = 3 + 2\cos(A)$

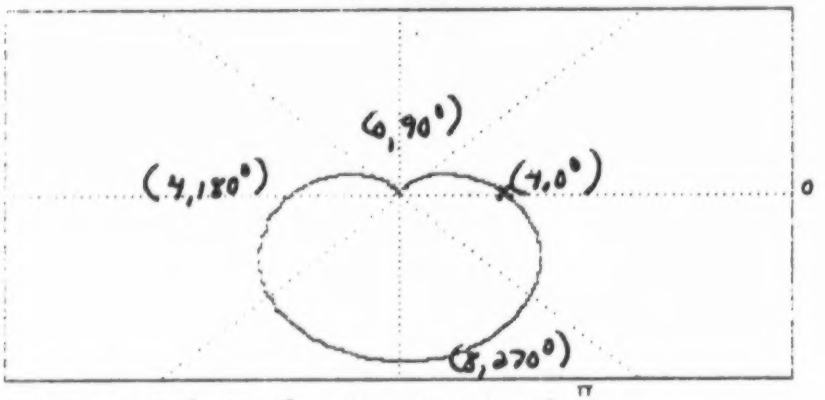


Lines at 0 to 2π in steps of $\frac{\pi}{4}$

2)

$R(A) = 4 - 4\,SIN(A)$

90° LINE

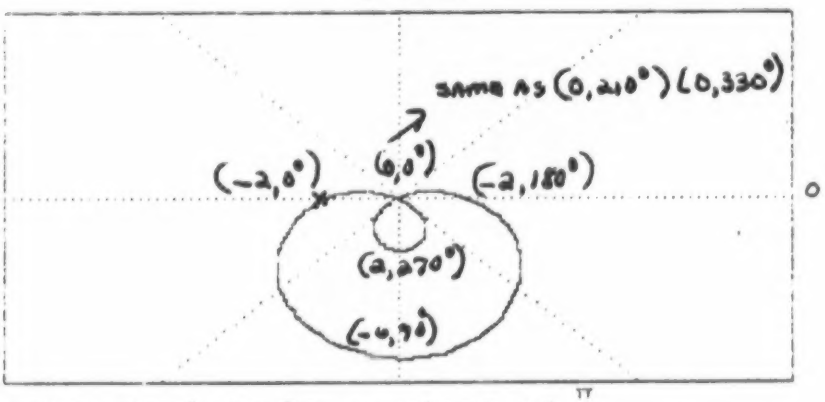


Lines at 0 to 2π in steps of $\frac{\pi}{4}$

3)

$R(A) = -2 - 4\,SIN(A)$

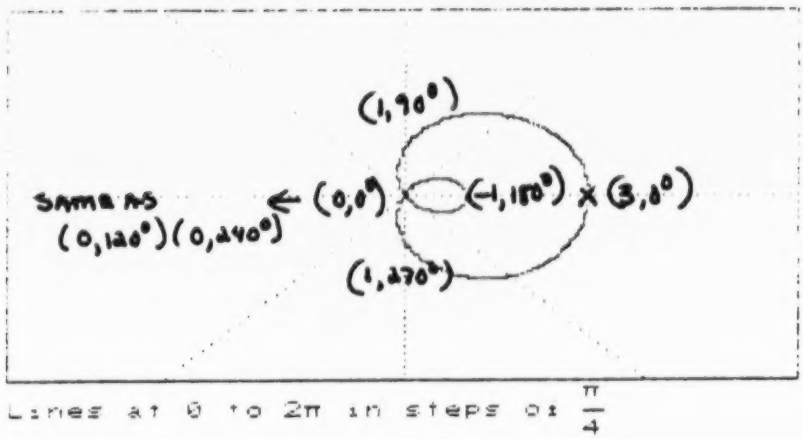
90° LINE



Lines at 0 to 2π in steps of $\frac{\pi}{4}$

4)

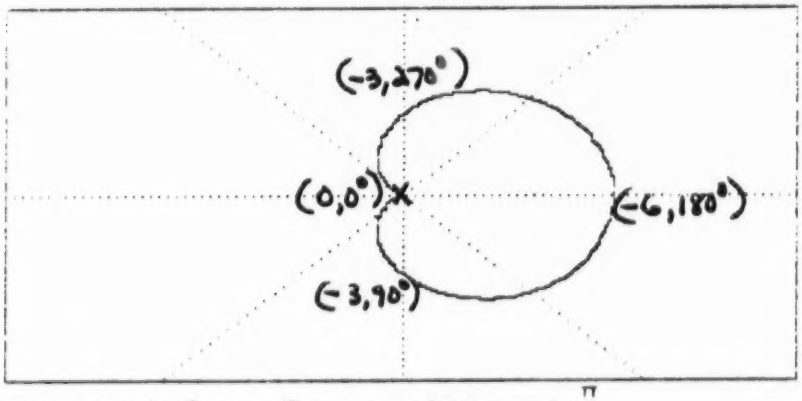
$R(A) = 1 + 2\cos(A)$



Lines at 0 to 2π in steps of $\frac{\pi}{4}$

5)

$R(A) = -3 + 3\cos(A)$

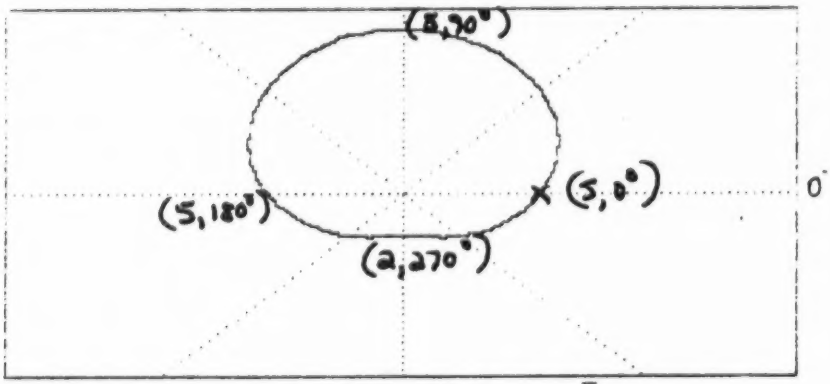


Lines at 0 to 2π in steps of $\frac{\pi}{4}$

90° LINE

6)

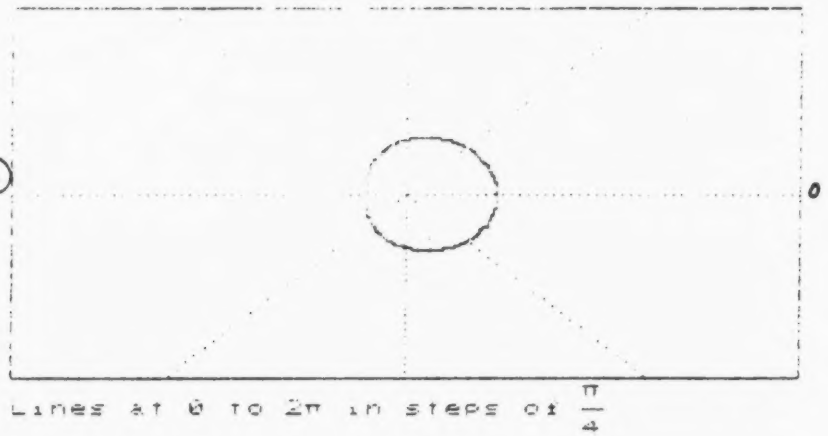
$R(A) = 5 + 3\sin(A)$



Lines at 0 to 2π in steps of $\frac{\pi}{4}$

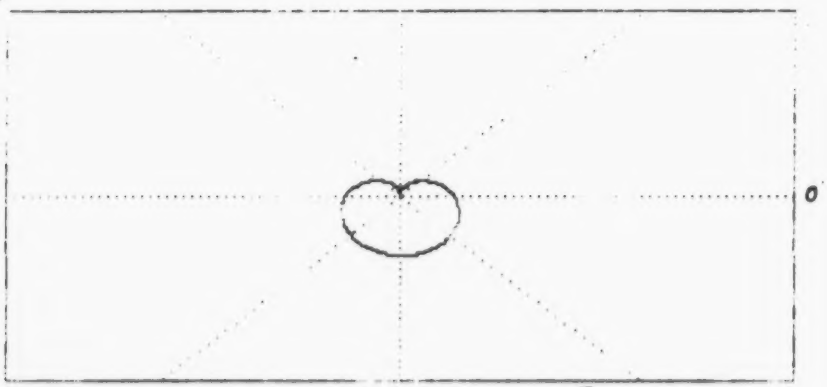
7)

$R^2(A) = 3 + 2\cos(A)$



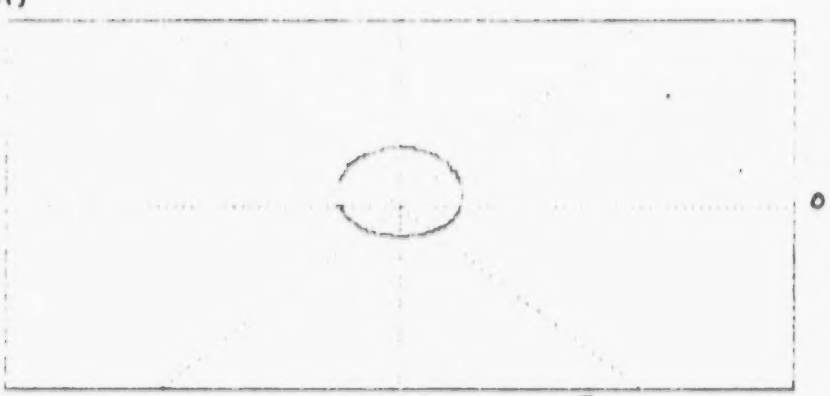
Lines at 0 to 2π in steps of $\frac{\pi}{4}$

$R^2(A) = 4 - 4\,SIN(A)$



Lines at 0 to 2π in steps of $\frac{\pi}{4}$

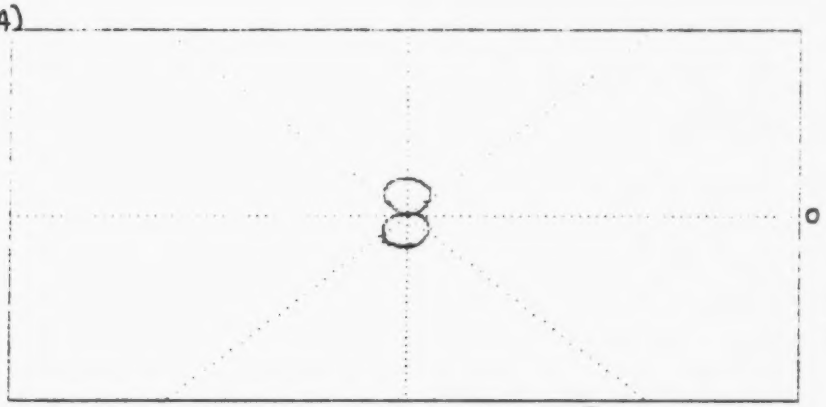
$R^2(A) = 5 + 3\,SIN(A)$



Lines at 0 to 2π in steps of $\frac{\pi}{4}$

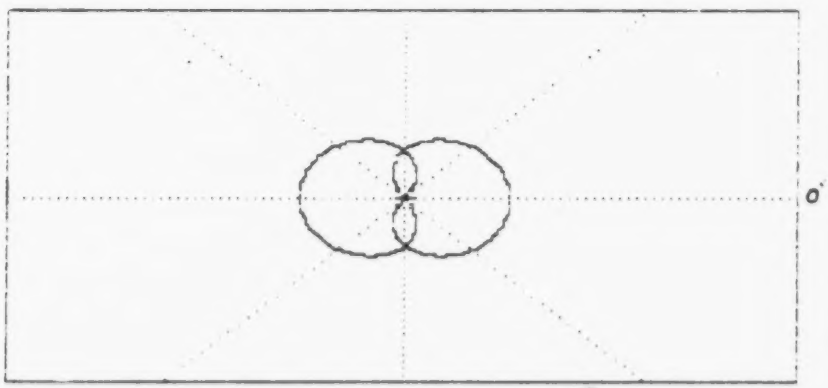
77

$R^2(A) = -2 - 4\,SIN(A)$



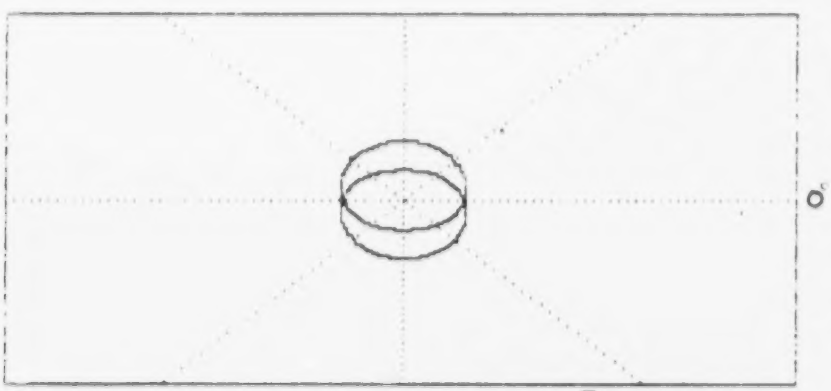
Lines at 0 to 2π in steps of $\frac{\pi}{4}$

$R^2(A) = 1 + 2\,COS(A)$



Lines at 0 to 2π in steps of $\frac{\pi}{4}$

$R^2(A) = -3 + 3\,COS(A)$



Lines at 0 to 2π in steps of $\frac{\pi}{4}$

8.4

TITLE: INTERACTIVE EXPERIMENTS IN CALCULUS
PUBLISHER: PRENTICE-HALL, INC.
ADDRESS: ENGLEWOOD CLIFFS
NJ, 07632
PRICE: $39.95
COMPUTER: APPLE II Plus/e/C
PIMMS' RATING: FAIR TO GOOD

PROGRAM IS APPROPRIATE FOR

ABILITY GROUP	: MIDDLE, HIGH
COURSE	: ADVANCED ALGEBRA, TRIGONOMETRY, ANALYTIC GEOMETRY, CALCULUS
FORMAT	: CLASS DEMO/INSTRUCTIONAL, DISCOVERY/LAB
CHARACTERISTICS	: DISK DEPENDENT TO SWITCH FROM PROGRAM TO PROGRAM BUT A STUDENT MANY WORK WITHIN A SINGLE PROGRAM WITHOUT THE DISK, GRAPHING UTILITY, DOCUMENTATION, PRINTER SET-UP PROGRAM, NO BACKUP DISKETTE
ADDITIONAL PERIPHERALS NEEDED	: COLOR MONITOR IS PREFERRED FOR THE GRAPHICS; .RINTER WITH GRAPHICS CAPABILITY CAN BE USED TO PRINT THE SCREEN GRAPHICS

Interactive Experiments in Calculus is designed to encourage exploration of definitions and concepts in Precalculus and Calculus ranging from limits to numerical integration to differential equations. Rational functions, parametric equations, infinite series, implicit functions, and inverse relations can be graphed with this package.

The program is easy to use. However, to fully appreciate its capabilities, it is recommended that the teacher first work on the tutorial for the graphing utility and then read through the documentation which accompanies each lesson (called an experiment). Explicit directions and a rationale are given for each lesson. The teacher may use any of the experiments in the package to illuminate and illustrate the objectives of the lesson or may use the experiments as a discovery learning activity before the concepts of the lesson are formally introduced. Five exploratory exercises accompany each experiment, but the teacher will have to develop related homework exercises.

If your printer has a Grappler interface, this program will allow the printing of screen graphics. This is a nice bonus when developing worksheets for your classes. However, the program also may be modified to change from bidirectional to unidirectional printing in order to get better resolution on the printout even though the unidirectional printing option is slower.

IF YOU PURCHASE THIS PROGRAM, CHECK OUT EACH OF THE LESSONS CAREFULLY BEFORE YOU CATALOG IT OR STAMP IT FOR YOUR INVENTORY. The first package I purchased worked fine, but a second package did not run certain programs (such as the numerical integration program). Prentice-Hall did accept the program for return but the waiting time and the correspondence was an annoyance. There is also a major error in the program in Experiments 2 and 9. Reference to the "area under the curve" is really the Riemann Sum. You have to decide whether or not the other capabilities of the program outweigh this error. These programs obviously were designed for use with positive-valued functions, but that does not excuse the error.

The program is versatile and relatively inexpensive. If you cannot afford more expensive Calculus packages, this one at least will get you started.

EXPLORING THE NATURE OF POLYNOMIAL FUNCTIONS

OBJECTIVES: To explore the relationship between the zeros of the first derivative function and the local (or relative) maximum or minimum values of the polynomial function.

To explore the relationship between the intervals over which the first derivative is positive/negative and the intervals over which the polynomial function is increasing/decreasing.

COURSE TITLE: Calculus

TITLE OF SOFTWARE PACKAGE NEEDED FOR THE LESSON: Interactive Experiments in Calculus (Prentice-Hall) or any graphing utility which allows more than one graph per set of axes and which allows for a change of scale.

TYPE OF ACTIVITY: Discovery/Lab
Students should be paired up, two to a computer.

SUGGESTED TIME: 1-1 1/2 Class Periods

PREREQUISITE SKILLS:
The student should:

- know the definitions of the zero of a function, local maximum or local minimum (local extrema), increasing function, decreasing function, polynomial function, linear function, and domain
- know how to take the first derivative of any polynomial function
- be able to find the zeros of a polynomial function.
- know how to use the "Graphing Utility" portion of Interactive Experiments in Calculus or a comparable graphing utility

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LABORATORY EXERCISES

EXPLORING THE NATURE OF POLYNOMIAL FUNCTIONS

For each of the following polynomial functions, find the first derivative; then graph the function and its first derivative on the same set of axes using Experiment 1, "Graphing Utility", of Interactive Experiments in Calculus or any other graphing utility which accepts two functions on the same graph.
If you have a printer which can print on-screen graphics, obtain a hard copy of each set of graphics. Answer the questions in Sections I and II for each polynomial function and its derivative before proceeding to the graphs of the next polynomial function and derivative function.

The boundary values for x and y are given for the first three problems. Experiment with the remaining two problems to obtain the "graphics window" which best displays the portion of the graphs needed to answer the questions. It is suggested that you use the option of placing reference dots on the screen. Enter 1 RETURN 1 when prompted for x spacing and y spacing.

f(x)	f'(x)	x: LL, UL	y: LL, UL
1) f(x) = .5*x^2+ x		-4,2	-2,4
2) f(x) = 3*x^3 - x		-2,2	-4,4
3) f(x) = x^3 - 3*x + 1		-2.5,2.5	-10,10
4) f(x) = -2*x - 3			
5) f(x) = -x^3 - 1			

I. Exploring Local Extrema

1. From the graphs determine all values of x for which the polynomial function, f(x), has a local maximum and a local minimum.

	Local Maximum	Local Minimum
1)		
2)		
3)		
4)		
5)		

2. Determine all values in the domain of f(x), (all values for x), for which the first derivative function, f'(x), equals zero.

1)

2)

3)

4)

5)

II. Exploring the Increasing or Decreasing Nature of Functions

1. From the graph find all values of x for which f(x) is an increasing function and for which f(x) is a decreasing function. Express your answer using interval notation.

f(x) increasing	f(x) decreasing

1)

2)

3)

4)

5)

2. Determine all values of x for which $f'(x)>0$ and for which $f'(x)<0$. Express your answer using interval notation.

$f'(x)\geq 0$	$f'(x)\leq 0$

1)

2)

3)

4)

5)

III. Questions for Discovery

1. Review your responses to questions 1 and 2 in Section I. For the functions 1-3, what appears to be the relationship between the value of the first derivative function and the local extrema of the polynomial function?

2. Is this same relationship true for functions 4 and 5?

   Discuss with your partner how functions 4 and 5 are different from functions 1-3 and different from each other.

3. Make a conjecture about the existence of local extrema for linear functions.

4. If the first derivative of a function equals zero, then the function always has a local maximum or a local minimum at the same values for x. True or false. Discuss your answer with your partner. Be able to justify your response to the rest of the class.

5. Review your responses to questions 1 and 2 from Section II. What appears to be the relationship between f'(x) and the increasing or decreasing nature of f(x)?

6. Do there appear to be any exceptions to the relationship you found in question #5 above for any of the polynomial functions you have graphed?

ANSWER KEY

LABORATORY EXERCISES

EXPLORING THE NATURE OF POLYNOMIAL FUNCTIONS

For each of the following polynomial functions, find the first derivative; then graph the function and its first derivative on the same set of axes using Experiment 1, "Graphing Utility", of Interactive Experiments in Calculus or any other graphing utility which accepts two functions on the same graph.
If you have a printer which can print on-screen graphics, obtain a hard copy of each set of graphics. Answer the questions in Sections I and II for each polynomial function and its derivative before proceeding to the graphs of the next polynomial function and derivative function.

The boundary values for x and y are given for the first three problems. Experiment with the remaining two problems to obtain the "graphics window" which best displays the portion of the graphs needed to answer the questions. It is suggested that you use the option of placing reference dots on the screen. Enter 1 RETURN 1 when prompted for x spacing and y spacing.

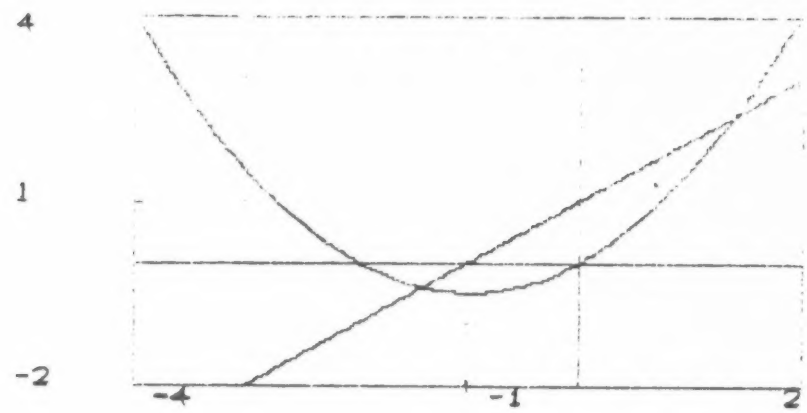
f(x)	f'(x)	x: LL, UL	y: LL, UL
1) f(x) = .5*x^2+ x	$f'(x) = x+1$	-4,2	-2,4
2) f(x) = 3*x^3 - x	$f'(x) = 9x^2-1$	-2,2	-4,4
3) f(x) = x^3 - 3*x + 1	$f'(x) = 3x^2-3$	-2.5,2.5	-10,10
4) f(x) = -2*x - 3	$f'(x) = -2$		
5) f(x) = -x^3 - 1	$f'(x) = -3x^2$		

I. Exploring Local Extrema

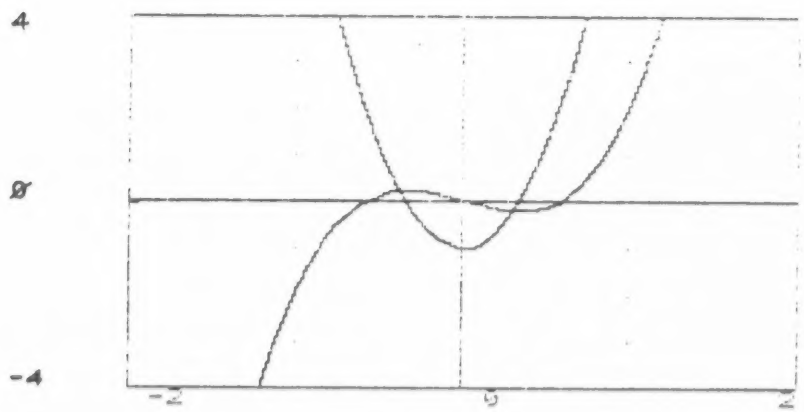
1. From the graphs determine all values of x for which the polynomial function, f(x), has a local maximum and a local minimum.

	Local Maximum	Local Minimum
1)	NONE	$x = -1$
2)	$x = -\frac{1}{3}$	$x = \frac{1}{3}$ (STUDENTS MIGHT ANSWER $x = -\frac{1}{4}, x = \frac{1}{4}$)
3)	$x = -1$	$x = 1$
4)	NONE	NONE
5)	NONE	NONE

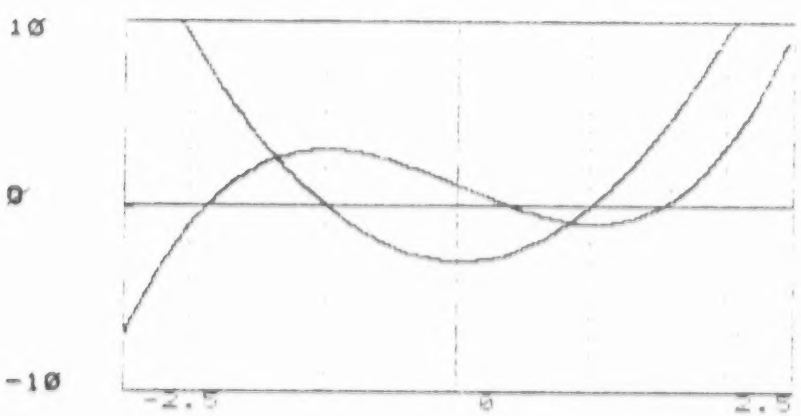
1) $y = .5x^2 + x$



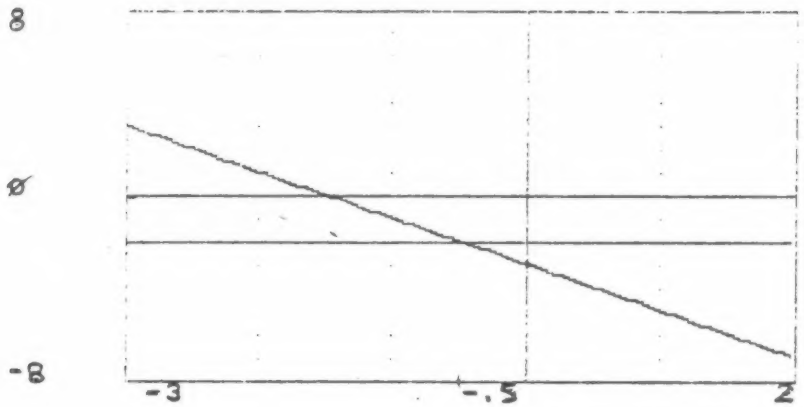
2) $y = 3x^3 - x$



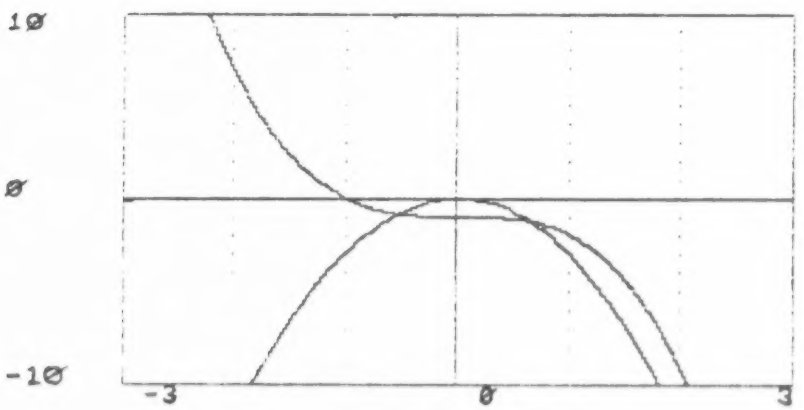
3) $y = x^3 - 3x + 1$



4) $y = -2x - 3$



5) $y = -x^3 - 1$



2. Determine all values in the domain of f(x), (all values for x), for which the first derivative function, f'(x), equals zero.

1) $x+1=0$ $\quad x=-1$

2) $9x^2-1=0$ $\quad x^2=\frac{1}{9}$ $\quad x=-\frac{1}{3}$ OR $x=\frac{1}{3}$

3) $3x^2-3=0$ $\quad x^2=1$ $\quad x=-1$ OR $x=1$

4) $-2\neq 0$

5) $3x^2=0$ $\quad x=0$

II. Exploring the Increasing or Decreasing Nature of Functions

1. From the graph find all values of x for which f(x) is an increasing function and for which f(x) is a decreasing function. Express your answer using interval notation.

	f(x) increasing	f(x) decreasing
1)	$x>-1$	$x<-1$
2)	$x<-\frac{1}{3}$ OR $x>\frac{1}{3}$: $(-\infty,-\frac{1}{3})\cup(\frac{1}{3},\infty)$	$-\frac{1}{3}<x<\frac{1}{3}$: $(-\frac{1}{3},\frac{1}{3})$
3)	$x<-1$ OR $x>1$: $(-\infty,-1)\cup(1,\infty)$	$-1<x<1$: $(-1,1)$
4)	NONE	$(-\infty,\infty)$
5)	NONE	$(-\infty,\infty)$

2. Determine all values of x for which f'(x)>0 and for which f'(x)<0. Express your answer using interval notation.

	f'(x)>0	f'(x)<0
1)	$x>-1$	$x<-1$
2)	$x<-\frac{1}{3}$ OR $x>\frac{1}{3}$: $(-\infty,-\frac{1}{3})\cup(\frac{1}{3},\infty)$	$-\frac{1}{3}<x<\frac{1}{3}$: $(-\frac{1}{3},\frac{1}{3})$
3)	$x<-1$ OR $x>1$: $(-\infty,-1)\cup(1,\infty)$	$-1<x<1$: $(-1,1)$
4)	NONE	$(-\infty,\infty)$
5)	NONE	$(-\infty,0)\cup(0,\infty)$

III. Questions for Discovery

1. Review your responses to questions 1 and 2 in Section I. For the functions 1-3, what appears to be the relationship between the value of the first derivative function and the local extrema of the polynomial function?

WHEN THE FIRST DERIVATIVE EQUALS ZERO, THE POLYNOMIAL FUNCTIONS 1-3 APPEAR TO HAVE LOCAL EXTREMA.

2. Is this same relationship true for functions 4 and 5?
FOR 4, THE FIRST DERIVATIVE NEVER EQUALS 0.
FOR 5, THE PREVIOUS STATEMENT DOES NOT APPEAR TO BE TRUE.
Discuss with your partner how functions 4 and 5 are different from functions 1-3 and different from each other. FUNCTION 4 IS LINEAR; FUNCTION 5 IS CUBIC.
FUNCTIONS 4 & 5 ARE BOTH DECREASING FUNCTIONS WHILE FUNCTIONS 1-3 SOMETIMES INCREASE AND SOMETIMES DECREASE.

3. Make a conjecture about the existence of local extrema for linear functions.
IF THE FIRST DERIVATIVE OF A FUNCTION IS ZERO AT A PARTICULAR VALUE OF THE DOMAIN, THEN A LOCAL EXTREMUM COULD OCCUR. IF THE FUNCTION IS ALWAYS INCREASING OR DECREASING, NO LOCAL EXTREMA WILL EXIST.

4. If the first derivative of a function equals zero, then the function always has a local maximum or a local minimum at the same values for x. True or false. Discuss your answer with your partner. Be able to justify your response to the rest of the class.

5. Review your responses to questions 1 and 2 from Section II. What appears to be the relationship between f'(x) and the increasing or decreasing nature of f(x)?
IF $f'(x)<0$, THE FUNCTION APPEARS TO ALWAYS DECREASE.
IF $f'(x)>0$, THE FUNCTION APPEARS TO ALWAYS INCREASE.

6. Do there appear to be any exceptions to the relationship you found in question #5 above for any of the polynomial functions you have graphed?
NO

HOMEWORK EXERCISES

I. For each of the following functions:
   a) find the first derivative, $f'(x)$.
   b) determine all values of x for which the function could possibly have a local maximum or a local minimum ($f'(x)=0$).
   c) name the intervals over which $f(x)$ is increasing; decreasing.
   d) sketch each $f(x)$ over the given interval and use the sketch to name any local extrema for each function and to verify your answer to c.

1) $f(x) = 2x^2 - 2x - 12$ $[-4,4]$

2) $f(x) = -\frac{x^3}{3} + 4x$ $[-4,4]$

3) $f(x) = x^3 + x^2 - 6x$ $[-10,10]$
(use the quadratic formula to solve $f'(x)=0$)

4) $f(x) = \frac{1}{3}x + 1$ $[-5,2]$

5) $f(x) = x^4 - 2x^2 + 1$ $[-3,3]$

II. The following functions are NOT polynomial functions.
For each of the following functions,
   a) Draw a sketch over the given interval.
   b) Find the first derivative of the function.
   c) Find all values of x for which the first derivative is zero or does not exist.
   d) Determine from the graph the values of x for each apparent local maximum or local minimum.
   e) Find all values of x for which the first derivative is positive and for which the first derivative is negative.
   f) Determine from the graph the values of x for which the function is increasing; decreasing.

1. $f(x) = |x - 5|$ $[-1,11]$

2. $f(x) = x\sqrt{4 - x2}$ $[-2,2]$

3. $f(x) = -\frac{1}{x^2}$ $[-4,4]$

   g) Compare your answers to c) and d) above. What statement can be made about the relationship of the value of the first derivative to the local extrema of the function?

   h) Compare your answers to e) and f) above. What statement can be made about the relationship of the value of the first derivative to the increasing or decreasing nature of a function?

Answer Key

HOMEWORK EXERCISES

I. For each of the following functions:
   a) find the first derivative, $f'(x)$.
   b) determine all values of x for which the function could possibly have a local maximum or a local minimum ($f'(x)=0$).
   c) name the intervals over which $f(x)$ is increasing; decreasing.
   d) sketch each $f(x)$ over the given interval and use the sketch to name any local extrema for each function and to verify your answer to c.

1) $f(x) = 2x^2 - 2x - 12$ $[-4,4]$

2) $f(x) = -\frac{x^3}{3} + 4x$ $[-4,4]$

3) $f(x) = x^3 + x^2 - 6x$ $[-10,10]$
(use the quadratic formula to solve $f'(x)=0$)

4) $f(x) = \frac{1}{3}x + 1$ $[-5,2]$

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II. The following functions are NOT polynomial functions.
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   d) Determine from the graph the values of x for each apparent local maximum or local minimum.
   e) Find all values of x for which the first derivative is positive and for which the first derivative is negative.
   f) Determine from the graph the values of x for which the function is increasing; decreasing.

1. $f(x) = |x - 5|$ $[-1,11]$
2. $f(x) = x\sqrt{4 - x^2}$ $[-2,2]$
3. $f(x) = \frac{1}{x^2}$ $[-4,4]$

   g) Compare your answers to c) and d) above. What statement can be made about the relationship of the value of the first derivative to the local extrema of the function?

   h) Compare your answers to e) and f) above. What statement can be made about the relationship of the value of the first derivative to the increasing or decreasing nature of a function?

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I.

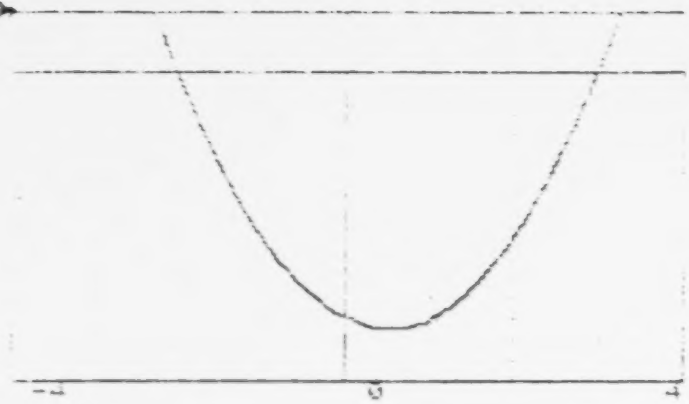
1) $y = 2x^2 - 2x - 12$

$4x - 2 = y'$

b) $x = \frac{1}{2}$

c) Dec: $x < \frac{1}{2}$
Inc: $x > \frac{1}{2}$

d) $x = \frac{1}{2}$, $y = -12.5$



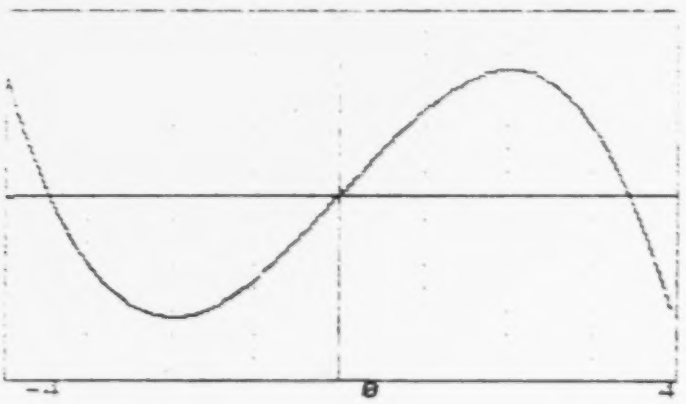
2) $y = -\frac{x^3}{3} + 4x$

a) $-x^2 + 4 = y'$

b) $x = -2$ or $x = 2$

c) Dec: $x < -2$ or $x > 2$
Inc: $-2 < x < 2$

d) $x = -2, y = -\frac{16}{3}$
$x = 2$ $y = \frac{16}{3}$



3) $y = x^3 + x^2 - 6x$

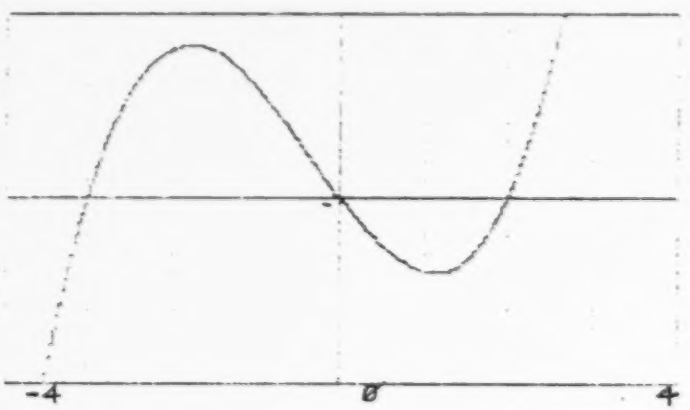
a) $y' = 3x^2 + 2x - 6$

b) $x = \left(-\frac{2}{3} \pm \frac{\sqrt{19}}{3}\right)$

$x = \frac{-5.4}{3}$ $x = \frac{3.4}{3}$

$x = -1.8$ $x = 1.1$

c)



Dec: $(-1.8, 1.1)$

Inc: $(-\infty, -1.8) \cup (1.1, \infty)$

d) $(-1.8, 8.2)$: $x = -1.8$, $y = 8.2$
$(1.1, -4.1)$: $x = 1.1$, $y = -4.1$

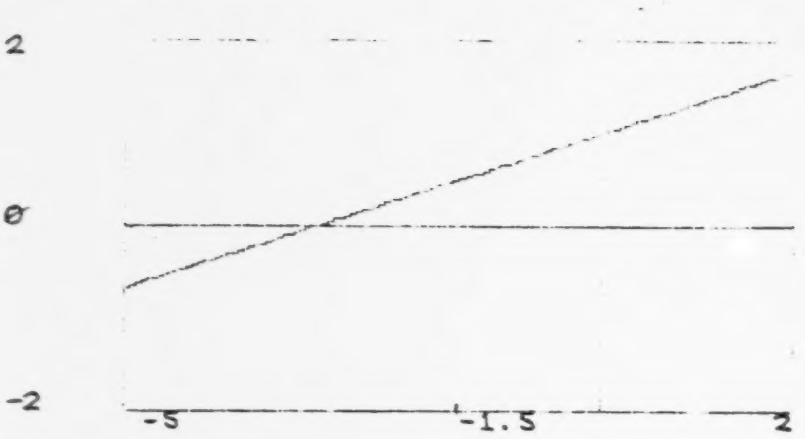
4) $y = \frac{1}{3}x + 1$

a) $y' = \frac{1}{3}$

b) NONE

c) INC: $(-\infty, \infty)$

d) NOWE



5) $y = x^4 - 2x^2 + 1$

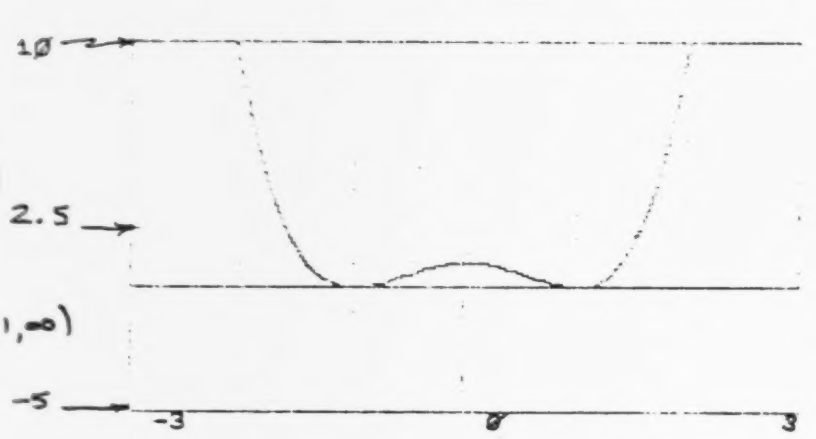
a) $y' = 4x^3 - 4x$

b) $x = 0, x = 1, x = -1$

c) DEC: $(-\infty, -1) \cup (0, 1)$

INC: $(-1, 0) \cup (1, \infty)$

d) $(-1, 0), (1, 0)$



II.

1. $y = |x-5|$

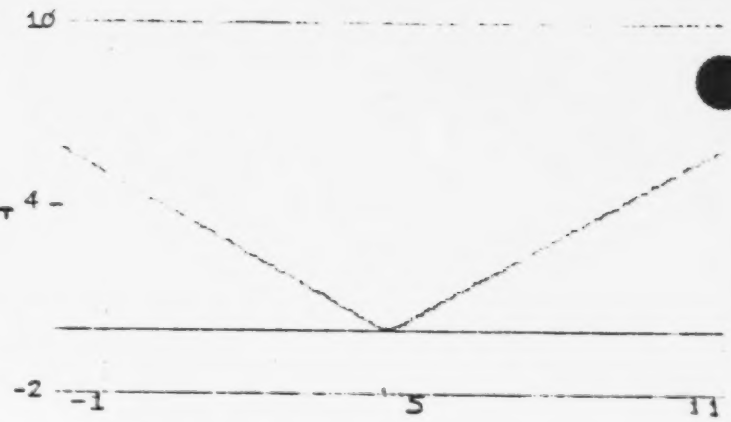
b) For $x>5$ $y'=1$
For $x<5$ $y'=-1$

c) $x=5$ f' does not exist

d) $x=5$

e) $(-\infty, 5)$, $f'<0$
$(5, \infty)$, $f'>0$

f) Dec: $(-\infty, 5)$
Inc: $(5, \infty)$



2. $y = x\sqrt{4-x^2}$

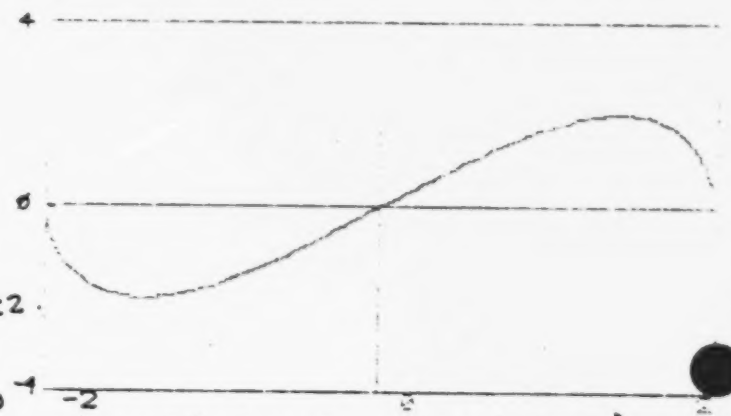
b) $y' = \dfrac{-2(x^2-2)}{\sqrt{4-x^2}}$

c) $f'=0$ at $x=\pm\sqrt{2}$
f' does not exist at $x=\pm 2$.

d) $x=-\sqrt{2}$ or $x=\sqrt{2}$

e) $(-2,\sqrt{2})$; $(\sqrt{2},2)$: $f'<0$
$(-\sqrt{2},\sqrt{2})$ $f'>0$

f) Dec: $(-2,-\sqrt{2}) \cup (\sqrt{2},2)$ Inc: $(-\sqrt{2},\sqrt{2})$



3. $y = \dfrac{1}{x^2}$

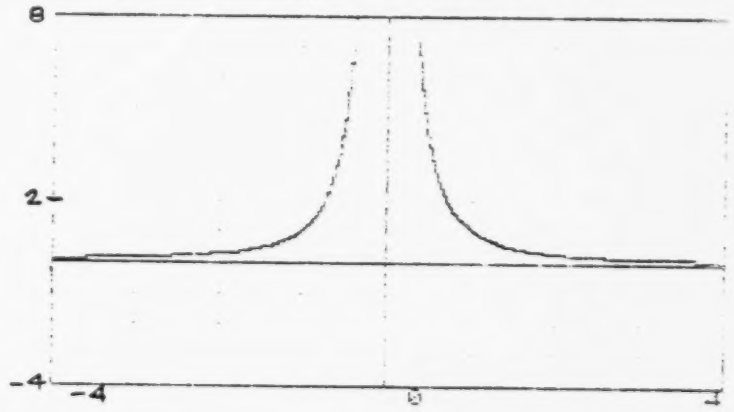
b) $y' = -2x^{-3}$

c) $x=0$ is not in the domain of f.

d) none

e) $f'>0$: $(-\infty, 0)$
$f'<0$: $(0, \infty)$

f) Dec: $(0, \infty)$
Inc: $(-\infty, 0)$



EXPLORING THE NATURE OF FUNCTIONS: POINTS OF INFLECTION AND CONCAVITY

OBJECTIVES: To explore the relationship between the zeros of the second derivative function and the points of inflection of the original function.

To explore the relationship between the intervals over which the second derivative is positive/negative and the intervals over which the polynomial function is concave up/concave down.

COURSE TITLE: Calculus

TITLE OF SOFTWARE PACKAGE NEEDED FOR THE LESSON: Interactive Experiments in Calculus or any graphing utility which allows more than one graph per set of axes and which allows for a change of scale.

TYPE OF ACTIVITY: Discovery/Lab
Students should be paired up, two to a computer.

SUGGESTED TIME: 1-1 1/2 Class Periods

PREREQUISITE SKILLS:
The student should:

- know the definitions of the zero of a function, concave up, concave down, point of inflection
- know how to find the second derivative of any algebraic function. (The teacher may use this activity to explore the nature of the curves of transcendental functions by changing the examples.)
- be able to find the zeros of a function
- know how to use the "Graphing Utility" portion of Interactive Experiments in Calculus or a comparable graphing utitliy

LABORATORY EXERCISES

EXPLORING THE NATURE OF FUNCTIONS: POINTS OF INFLECTION AND CONCAVITY

For each of the following functions, find the second derivative then graph the function and its second derivative on the same set of axes using Experiment 1, "Graphing Utility", of Interactive Experiments in Calculus. If you have a printer which can print the on-screen graphics, obtain a hard copy of each set of graphs.

Answer the questions in Sections I and II for each function and its second derivative before proceeding to the graphs of the next pair of functions.

The boundary values for x and y are given for the first two problems. Experiment with the remaining four problems to obtain the "graphics window" which best displays the portion of the graphs needed to answer the questions. It is suggested that you use the option of placing reference dots on the screen. Enter .5 RETURN .5 RETURN when prompted for x spacing and y spacing.

f(x)	f"(x)	x:LL,UL	y:LL,UL
1) f(x) = .5*x^2 + x		-4,2	-2,4
2) f(x) = 3*x^3 - x		-2,2	-4,4
3) f(x) = x^3 - 3*x + 1			
4) f(x) = -x^3 - 1			
5) f(x) = x^4 - 2*x^2 + 1			
6) f(x) = 1/(x^2)			

I. Exploring Points of Inflection

1. From the graphs determine all values of x for which each function, f(x), has a point of inflection.

   1)

   2)

   3)

   4)

   5)

   6)

2. Determine all values in the domain of f(x) for which the second derivative function, f"(x), equals zero.

1)

2)

3)

4)

5)

6)

II. Exploring the Concavity of Functions

1. From the graph, determine all values of x for which f(x) is concave up and concave down. Express your answer using interval notation.

	Concave Up	Concave Down
1)		
2)		
3)		
4)		
5)		
6)		

2. Determine all values of x for which f"(x)>0 and for which f"(x)<0. Express your answer using interval notation.

	f"(x)>0	f"(x)<0
1)		
2)		
3)		
4)		
5)		
6)		

III. Questions for Discovery

1.a)Review your responses to questions 1 and 2 in Section I. What appears to be the relationship between the value of the second derivative function and the point or points of inflection of the original function?

b)How is function 6 different from the other functions?

2.a)Find the first derivative for function 4, f(x) = -x3 - 1.

b)Find all values of x for which the first derivative equals zero.

c)Using the graphing utility, draw the graphs of the function, its first derivative and its second derivative on the same set of axes.

d)Does the original function have local extrema at all values of x for which the first derivative equals zero?

e)What is the value of the second derivative at this point?

f)What is the name for this type of point?

g)Using the graphing utility, draw the graphs of f(x) = x^5, f'(x), and f"(x) on the same set of axes.

h)Use the graph to name the point of inflection.

i)Find the value of the first and second derivatives of f(x) for this value of x.

j)Use your answers to the questions above to discuss the statement: If the first derivative of a function equals zero, then the function MAY have a local extrema at that point. Make a conjecture that will guarantee that a local extrema is present when f'(x) equals zero.

3) Review your responses to questions 1 and 2 from Section II. What appears to be the relationship between f"(x) and the concavity of the function?

4) Describe a procedure for testing the concavity of a function. (Keep in mind a function like function 6.)

5) Can we speak about the concavity of linear functions or functions of the form $|mx + b|$? Discuss this question with your partner and be able to defend your conclusions to the rest of the class.

ANSWER KEY

LABORATORY EXERCISES

EXPLORING THE NATURE OF FUNCTIONS:
POINTS OF INFLECTION AND CONCAVITY

For each of the following functions, find the second derivative then graph the function and its second derivative on the same set of axes using Experiment 1, "Graphing Utility", of Interactive Experiments in Calculus. If you have a printer which can print the on-screen graphics, obtain a hard copy of each set of graphs.

Answer the questions in Sections I and II for each function and its second derivative before proceeding to the graphs of the next pair of functions.

The boundary values for x and y are given for the first two problems. Experiment with the remaining four problems to obtain the "graphics window" which best displays the portion of the graphs needed to answer the questions. It is suggested that you use the option of placing reference dots on the screen. Enter .5 RETURN .5 RETURN when prompted for x spacing and y spacing.

f(x)	f"(x)	x:LL,UL	y:LL,UL
1) f(x) = .5*x^2 + x	$f''(x)=1$	-4,2	-2,4
2) f(x) = 3*x^3 - x	$f''(x)=18x$	-2,2	-4,4
3) f(x) = x^3 - 3*x + 1	$f''(x)=6x$		
4) f(x) = -x^3 - 1	$f''(x)=-6x$		
5) f(x) = x^4 - 2*x^2 + 1	$f''(x)=12x^2-4$		
6) f(x) = 1/(x^2)	$f''(x)=6x^{-4}$		

I. Exploring Points of Inflection

1. From the graphs determine all values of x for which each function, f(x), has a point of inflection.

   1) NONE

   2) $x=0$

   3) $x=0$

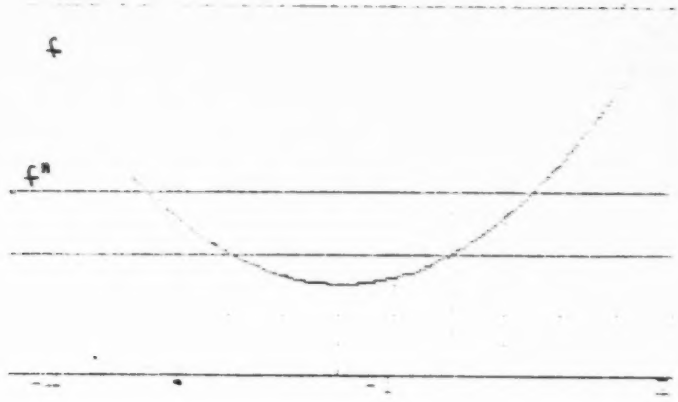
   4) $x=0$

   5) $x \doteq -.6$ or $x \doteq .6$

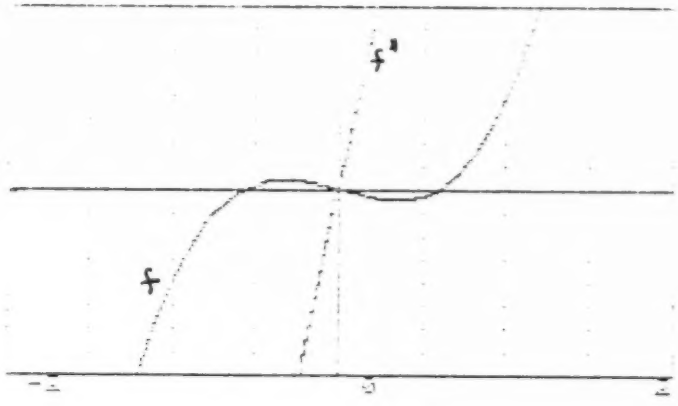
   6) NONE

ANSWER KEY

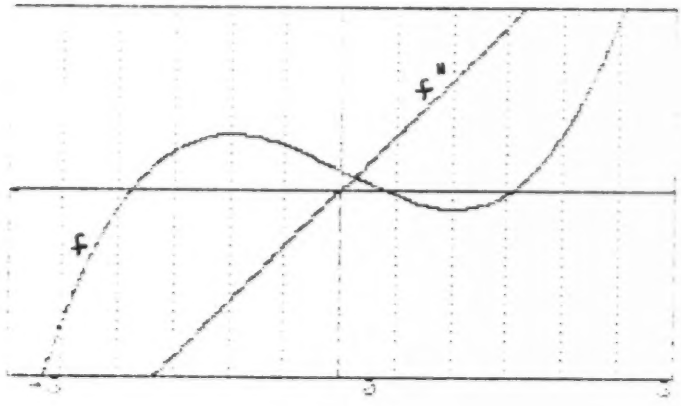
I. 2. 1) $f(x) = .5x^2 + x$ $\quad f''(x) = 1$



2) $f(x) = 3x^3 - x$ $\quad f''(x) = 18x$

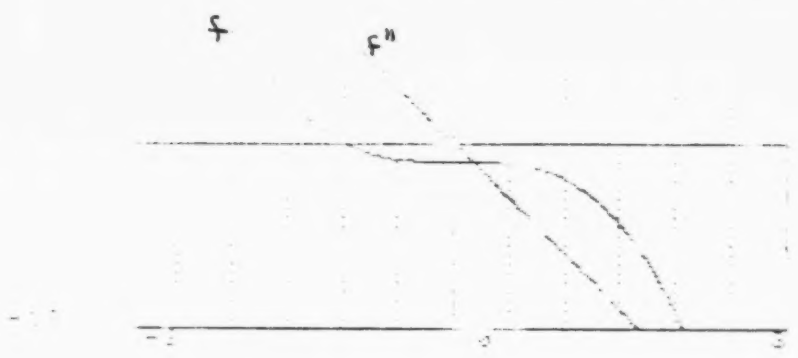


3) $f(x) = x^3 - 3x + 1$ $\quad f''(x) = 6x$

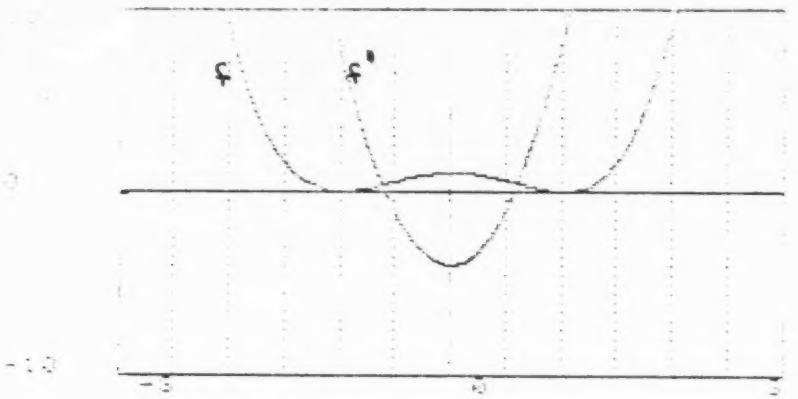


I. 2. 4)

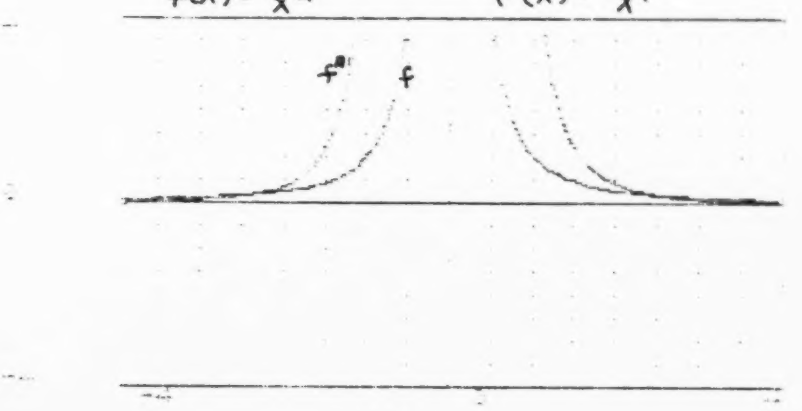
$f(x) = -x^3 - 1$ $\quad f''(x) = -6x$



5) $f(x) = x^4 - 2x^2 + 1$ $\quad f''(x) = 12x^2 - 4$



6) $f(x) = \frac{1}{x^2}$ $\quad f''(x) = \frac{6}{x^4}$



2. Determine all values in the domain of f(x) for which the second derivative function, f"(x), equals zero.

1) $1 \neq 0$ NONE

2) $18x = 0$ WHEN $x = 0$

3) $6x = 0$ WHEN $x = 0$

4) $-6x = 0$ WHEN $x = 0$

5) $12x^{\frac{1}{2}} - 4 = 0$ WHEN $x = \pm\frac{\sqrt{3}}{3}$ (OR $x \approx \pm .58$)

6) $6x^{-4} \neq 0$ NONE

II. Exploring the Concavity of Functions

1. From the graph, determine all values of x for which f(x) is concave up and concave down. Express your answer using interval notation.

	Concave Up	Concave Down
1)	$(-\infty, \infty)$	NONE
2)	$(0, \infty)$	$(-\infty, 0)$
3)	$(0, \infty)$	$(-\infty, 0)$
4)	$(-\infty, 0)$	$(0, \infty)$
5)	$(-\infty, -.6) \cup (.6, \infty)$	$(-.6, .6)$
6)	$(-\infty, 0) \cup (0, \infty)$	NONE

2. Determine all values of x for which f"(x)>0 and for which f"(x)<0. Express your answer using interval notation.

	f"(x)>0	f"(x)<0
1)	$(-\infty, \infty)$	NONE
2)	$(0, \infty)$	$(-\infty, 0)$
3)	$(0, \infty)$	$(-\infty, 0)$
4)	$(-\infty, 0)$	$(0, \infty)$
5)	$(-\infty, -\frac{\sqrt{3}}{3}) \cup (\frac{\sqrt{3}}{3}, \infty)$	$(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3})$
6)	$(-\infty, 0) \cup (0, \infty)$	NONE

III. Questions for Discovery

1.a) Review your responses to questions 1 and 2 in Section I. What appears to be the relationship between the value of the second derivative function and the point or points of inflection of the original function?

THE POINT OF INFLECTION APPEARS TO OCCUR WHEN THE VALUE OF THE SECOND DERIVATIVE EQUALS ZERO.

b) How is function 6 different from the other functions?

THE DOMAIN HAS A RESTRICTION, $x \neq 0$.

2.a) Find the first derivative for function 4, $f(x) = -x3 - 1$.

$f'(x) = -3x^2$

b) Find all values of x for which the first derivative equals zero.

$-3x^2 = 0$ WHEN $x = 0$

c) Using the graphing utility, draw the graphs of the function, its first derivative and its second derivative on the same set of axes. See next page.

d) Does the original function have local extrema at all values of x for which the first derivative equals zero?

NO

e) What is the value of the second derivative at this point?

0

f) What is the name for this type of point?

A POINT OF INFLECTION

g) Using the graphing utility, draw the graphs of $f(x) = x^5$, $f'(x)$, and $f''(x)$ on the same set of axes. See next page.

$f'(x) = 5x^4$ $f''(x) = 20x^3$

h) Use the graph to name the point of inflection.

(0,0)

i) Find the value of the first and second derivatives of f(x) for this value of x.

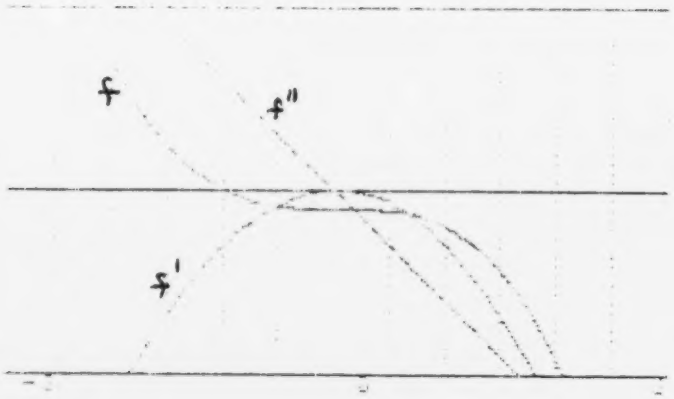
$f'(0) = 0$ $f''(0) = 0$

j) Use your answers to the questions above to discuss the statement: If the first derivative of a function equals zero, then the function MAY have a local extrema at that point. Make a conjecture that will guarantee that a local extrema is present when $f'(x)$ equals zero.

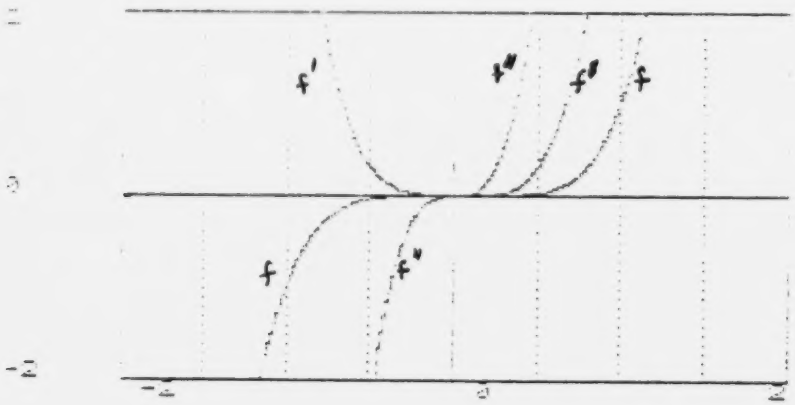
(TEACHER DIRECTED DISCUSSION. A COMPARISON OF THE FIRST DERIVATIVE TEST AND THE SECOND DERIVATIVE TEST IS APPROPRIATE.)

III

2.c) $f(x) = -x^3 - 1$ $\quad f'(x) = -3x^2$ $\quad f''(x) = -6x$



2.g) $f(x) = x^5$ $\quad f'(x) = 5x^4$ $\quad f''(x) = 20x^3$



3) Review your responses to questions 1 and 2 from Section II. What appears to be the relationship between f"(x) and the concavity of the function?

IF $f''(x) > 0$, THE FUNCTION APPEARS TO BE CONCAVE UP.
IF $f''(x) < 0$, THE FUNCTION APPEARS TO BE CONCAVE DOWN.

4) Describe a procedure for testing the concavity of a function. (Keep in mind a function like function 6.)

FIND ALL VALUES OF X FOR WHICH $f''(x) = 0$ OR DOES NOT EXIST. CHECK THE INTERVALS FORMED BY THESE VALUES OF X TO DETERMINE IF $f''(x)$ IS GREATER THAN OR LESS THAN ZERO. (THE FUNCTION MUST BE DIFFERENTIABLE OVER THESE INTERVALS.)

5) Can we speak about the concavity of linear functions or functions of the form $|mx + b|$? Discuss this question with your partner and be able to defend your conclusions to the rest of the class.

SINCE $f''(x) = 0$ FOR ALL X, THE PROCEDURE DESCRIBED IN QUESTION #4 DOES NOT APPLY. THUS, CONCAVITY CANNOT BE DISCUSSED.

HOMEWORK EXERCISES

I. For each of the functions listed below:
   a) find the second derivative, f"(x).
   b) determine all values of x for which the function has a point of inflection, i.e., when f"(x) = 0 or does not exist.
   c) name the intervals over which f(x) is concave up; concave down.
   d) find its first derivative, f'(x).
   e) determine all values of x for which the function may have a local extremum, i.e., when f'(x) = 0 or where f'(x) does not exist.
   f) name the intervals over which f(x) is increasing; decreasing.
   g) find the function's value for each point of inflection and each local extremum.
   h) using all of the above information, SKETCH the graph of f(x) without plotting points.
   i) The zeros of the the function would be helpful in sketching the graph of f. If possible, find the zeros of f and refine your graph.

1) $f(x) = \frac{-x3}{3} + 4x$

2) $f(x) = -2x^4 + 4x^2$

3) $f(x) = -x^5 + 5x^3$

4) $f(x) = |\, 2x^2 - 2 \,|$

5) $f(x) = 1/x$

II. For each of the functions listed below:
   a) find f'(t), f"(t), f'''(t).
   b) find all values of t for which the first derivative function may have local extrema, i.e., when f"(t) = 0 or does not exist.
   c) use the third derivative to verify the local extrema.
   d) find all values of t for which the first derivative function is increasing; decreasing, i.e. when f"(t)>0 or f"(t)<0.
   e) find all points of inflection for the first derivative function.
   f) name the intervals over which the first derivative function is concave up; concave down, i.e. when f'''(t)>0 or f'''(t)<0.
   g) using the above information, SKETCH the first derivative function.

1) $f(t) = t^4 - 2t^2 + 1$

2) $f(t) = 2t^3 - 6t^5$

Answer Key

HOMEWORK EXERCISES

I. For each of the functions listed below:
   a) find the second derivative, f"(x).
   b) determine all values of x for which the function has a point of inflection, i.e., when f"(x) = 0 or does not exist.
   c) name the intervals over which f(x) is concave up; concave down.
   d) find its first derivative, f'(x).
   e) determine all values of x for which the function may have a local extremum, i.e., when f'(x) = 0 or where f'(x) does not exist.
   f) name the intervals over which f(x) is increasing; decreasing.
   g) find the function's value for each point of inflection and each local extremum.
   h) using all of the above information, SKETCH the graph of f(x) without plotting points.
   i) The zeros of the the function would be helpful in sketching the graph of f. If possible, find the zeros of f and refine your graph.

   1) $f(x) = \frac{-x3}{3} + 4x$

   2) $f(x) = -2x^4 + 4x^2$

   3) $f(x) = -x^5 + 5x^3$

   4) $f(x) = |\, 2x^2 - 2 \,|$

   5) $f(x) = 1/x$

II. For each of the functions listed below:
   a) find f'(t), f"(t), f'''(t).
   b) find all values of t for which the first derivative function may have local extrema, i.e., when f"(t) = 0 or does not exist.
   c) use the third derivative to verify the local extrema.
   d) find all values of t for which the first derivative function is increasing; decreasing, i.e. when f"(t)>0 or f"(t)<0.
   e) find all points of inflection for the first derivative function.
   f) name the intervals over which the first derivative function is concave up; concave down, i.e. when f'''(t)>0 or f'''(t)<0.
   g) using the above information, SKETCH the first derivative function.

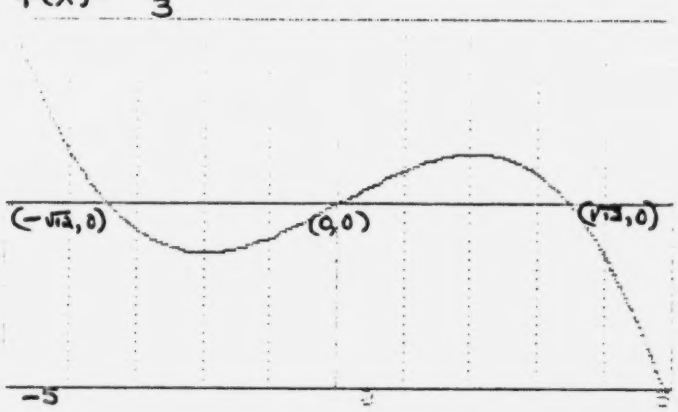
   1) $f(t) = t^4 - 2t^2 + 1$

   2) $f(t) = 2t^3 - 6t^5$

HOMEWORK EXERCISES

I.

① $f(x) = -\frac{x^3}{3} + 4x$



a) $f''(x) = -2x$

b) $x = 0$

c) CU: $(-\infty, 0)$
CD: $(0, \infty)$

d) $f'(x) = -x^2 + 4$

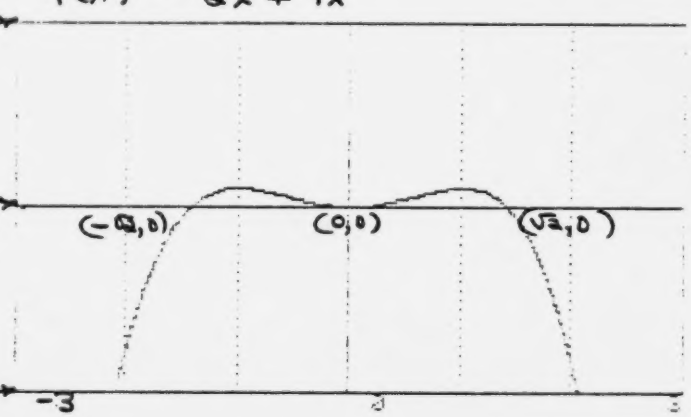
e) $x = -2$ or $x = 2$

f) DEC: $(-\infty, -2) \cup (2, \infty)$
INC: $(-2, 2)$

g) PI: $(0, 0)$

L. EXTREMA: $(-2, -\frac{16}{3})$ $(2, \frac{16}{3})$

② $f(x) = -2x^4 + 4x^2$



a) $f''(x) = -24x^2 + 8$

b) $x = -\frac{\sqrt{3}}{3}$ or $x = \frac{\sqrt{3}}{3}$

c) CU: $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$
CD: $\left(-\infty, -\frac{\sqrt{3}}{3}\right) \cup \left(\frac{\sqrt{3}}{3}, \infty\right)$

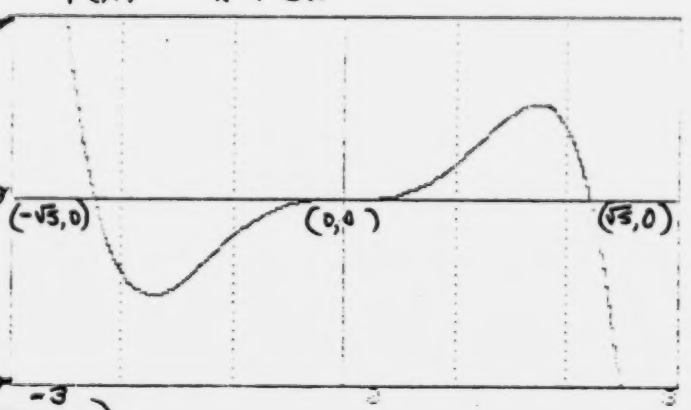
d) $f'(x) = -8x^3 + 8x$

e) $x = 0$, $x = -1$, or $x = 1$

f) DEC: $(-1, 0) \cup (1, \infty)$
INC: $(-\infty, -1) \cup (0, 1)$

g) PI: $\left(-\frac{\sqrt{3}}{3}, \frac{10}{9}\right)$ $\left(\frac{\sqrt{3}}{3}, \frac{10}{9}\right)$ L. EXTREMA: $(-1, 2)$ $(0, 0)$ $(1, 2)$

③ $f(x) = -x^5 + 5x^3$



a) $f''(x) = -20x^3 + 30x$

b) $x = 0$, $x = -\frac{\sqrt{6}}{2}$, $x = \frac{\sqrt{6}}{2}$

c) CU: $\left(-\infty, -\frac{\sqrt{6}}{2}\right) \cup \left(0, \frac{\sqrt{6}}{2}\right)$
CD: $\left(-\frac{\sqrt{6}}{2}, 0\right) \cup \left(\frac{\sqrt{6}}{2}, \infty\right)$

d) $f'(x) = -5x^4 + 15x^2$

e) $x = 0$, $x = -\sqrt{3}$, $x = \sqrt{3}$

f DEC: $(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$
INC: $(-\sqrt{3}, \sqrt{3})$

g) PI: $(0, 0)$ $\left(-\frac{\sqrt{6}}{2}, -8.6\right)$ $\left(\frac{\sqrt{6}}{2}, 8.6\right)$ L. EXTREMA: $(-\sqrt{3}, -6\sqrt{3})$ $(\sqrt{3}, 6\sqrt{3})$

(4)

$f(x) = |2x^2 - 2|$

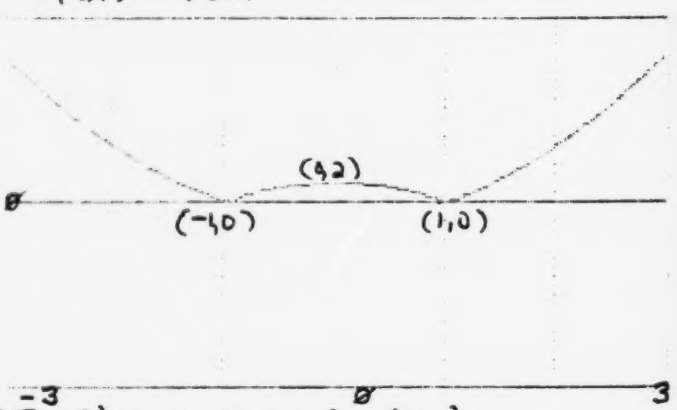
a) $f(x) = \begin{cases} 4 \text{ for } x < -1 \text{ or } x > 1 \\ -4 \text{ for } -1 < x < 1 \end{cases}$

b) $f''(x)$ NEVER $= 0$. DOESN'T EXIST AT $x = -1$ OR $x = 1$

c) CU: $(-\infty, -1) \cup (1, \infty)$
CD: $(-1, 1)$

d) $f'(x) = \begin{cases} 4x \text{ IF } x < -1 \text{ or } x > 1 \\ -4x \text{ IF } -1 < x < 1 \end{cases}$

e) $x = -1$ OR $x = 1$ DOES NOT EXIST
$x = 0$ $f'(x) = 0$



f) DEC: $(-\infty, -1) \cup (0, 1)$
INC: $(-1, 0) \cup (1, \infty)$

g) PI: $(-1, 0), (1, 0)$
L. EXTREMA: $(-1, 0), (0, 2), (1, 0)$

(5)

a) $f''(x) = 2x^{-3}$

b) DOES NOT EXIST AT $x = 0$

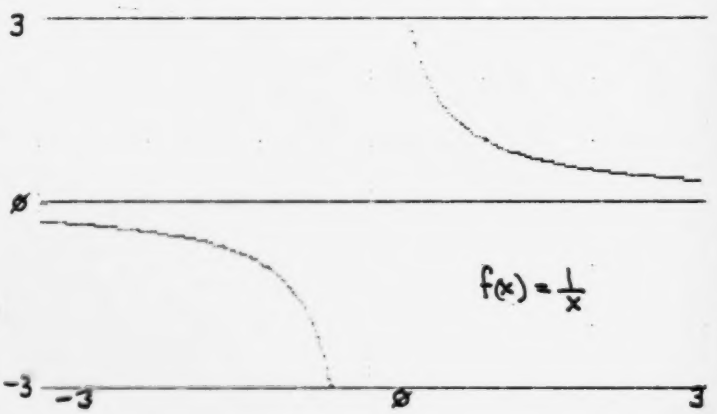
c) CU: $(0, \infty)$
CD: $(-\infty, 0)$

d) $f'(x) = -x^{-2}$

e) DOES NOT EXIST AT $x = 0$

f) DEC: $(-\infty, 0) \cup (0, \infty)$

g) PI: NONE
L. EXTREMA: NONE



II

1)

a) $f'(t) = 4t^3 - 4t$

$f''(t) = 12t^2 - 4$

$f'''(t) = 24t$

b) $f'(t) = 0$

$12t^2 - 4 = 0$

$t = \pm\frac{\sqrt{3}}{3}$

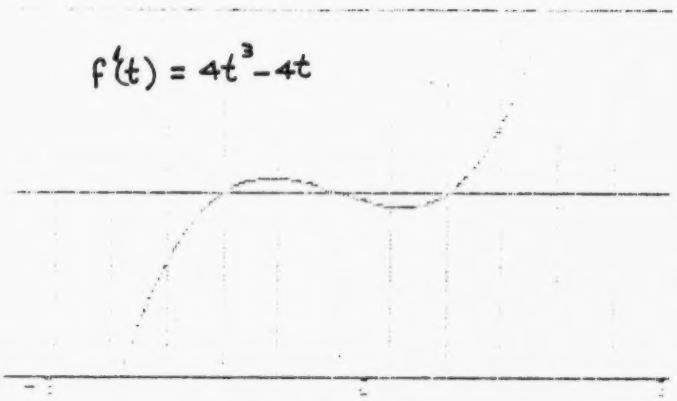
c) $f''\left(-\frac{\sqrt{3}}{3}\right) < 0$ ∴ LOCAL MAX

$f''\left(+\frac{\sqrt{3}}{3}\right) > 0$ ∴ LOCAL MIN

d) INC: $\left(-\infty, -\frac{\sqrt{3}}{3}\right) \cup \left(\frac{\sqrt{3}}{3}, \infty\right)$

DEC: $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$

g)

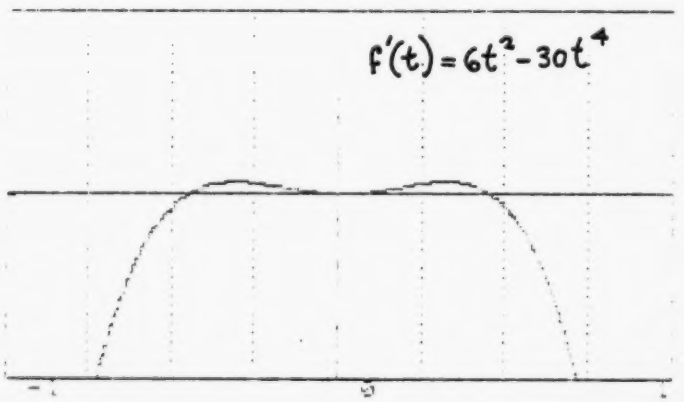


e) $(0,0)$ f) $(-\infty, 0)$ CONCAVE DOWN

$(0, \infty)$ CONCAVE UP

2)

g)



a) $f'(t) = 6t^2 - 30t^4$

$f''(t) = 12t - 120t^3$

$f'''(t) = 12 - 360t^2$

b) $f''(t) = 0$

$12t - 120t^3 = 0$

$t = 0$ OR $t = \pm\frac{\sqrt{10}}{10}$

c) $f''(0) > 0$ ∴ LOCAL MIN

$f'''\left(\pm\frac{\sqrt{10}}{10}\right) < 0$ ∴ LOCAL MAX

d) INC: $\left(-\infty, -\frac{\sqrt{10}}{10}\right) \cup \left(0, \frac{\sqrt{10}}{10}\right)$

DEC: $\left(-\frac{\sqrt{10}}{10}, 0\right) \cup \left(\frac{\sqrt{10}}{10}, \infty\right)$

e) $f''(t) = 0 \quad 12 - 360t^2 = 0$

$t = \pm\frac{\sqrt{30}}{30}$ PI $\left(-\frac{\sqrt{30}}{30}, \frac{1}{6}\right)\left(\frac{\sqrt{30}}{30}, \frac{1}{6}\right)$

f) CU: $\left(-\frac{\sqrt{30}}{30}, \frac{\sqrt{30}}{30}\right)$

CD: $\left(-\infty, -\frac{\sqrt{30}}{30}\right) \cup \left(\frac{\sqrt{30}}{30}, \infty\right)$

TECHNIQUES OF NUMERICAL INTEGRATION

OBJECTIVES: To explore, via use of rectangular approximation, the concept of the area under a curve for a continuous, nonnegative function by use of rectangular approximation.*

To explore the meaning of the limit of the sum of these rectangular approximations as the number of subintervals increases.

(Optional) To compare the accuracy of the rectangular approximation method with the Midpoint Rule, Trapezoidal Rule, and Simpson's Rule.

COURSE TITLE: Calculus

TITLE OF SOFTWARE PACKAGE NEEDED FOR THE LESSON: Interactive Experiments in Calculus.

TYPE OF ACTIVITY: Class Demo/Instructional

SUGGESTED TIME: 1 Class Period

PREREQUISITE SKILLS:
The student should:

- understand the meaning of the area under a curve over a given closed interval
- be able to distinguish if a rectangle is circumscribed about a curve or inscribed within it
- be familiar with summation notation
- be familiar with infinite limits.

* Enter only continuous, nonnegative functions for this exercise. The program actually finds the Riemann Sum for a function f over a closed interval $[a,b]$ even though it is labeled as the area under the curve.

TECHNIQUES OF NUMERICAL INTEGRATION

NOTES FOR THE TEACHER:

If you have not used the program Interactive Experiments In Calculus before, it is recommended that you read through the tutorial on the graphing utility, pages 2 through 19. If you are short of time, just read pages 2 through 9.

Have available two computers to use for your demonstration. Prior to the class session load the program Interactive Experiments In Calculus. Using the right arrow key scroll down the list of options in the main menu until you come to "Numerical Integration." Hit return and wait until the program is loaded. If you do not have two program packages available, the second machine may be loaded in the same way using the same diskette. Remember, you may not run another program from the main menu unless the program diskette is in the disk drive, but you may run the numerical integration part of the program repeatedly without having the program diskette in the drive.

1) Enter the function f(x) = x ^ 2, ($f(x) = x^2$) in the first machine.

2) Enter zero as the lower bound for x and 4 as the upper bound for x. Enter -1 as the lower bound for y and 16 as the upper bound for y. (Note: The number zero must be within the y-axis range.)

3) Enter 4 as the number of subintervals, (n = 4). The program calls the approximating figures which will be drawn "strips."

4) Select the left edge option, L. This option uses approximating rectangles with the height of the rectangle always the left dimension of each strip.

5) Have students record this very rough estimate of the area under the curve from a = 0 to b = 4.

6) Before continuing, introduce the summation notation, $\sum_{i=0}^{3} f(x_i)\Delta x$

its expansion, $f(x_0)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x$

the notation for the length of the subinterval,
x = (b- a)/n and its value, $\frac{4-0}{4} = 1$

and the value of the expression for the given example.

$$f(0)(1) + f(1)(1) + f(2)(1) + f(3)(1)$$
$$0 + 1 + 4 + 9 = 14$$

7) Repeat steps 1 through 6 on the second machine using all the same parameters except letting the number of subintervals increase from n = 4 to n = 10.

$$\sum_{i=0}^{9} f(x_i)\Delta x = f(x_0)\Delta x + f(x_1)\Delta x + \ldots + f(x_9)\Delta x =$$
$$f(0)(.4) + f(.4)(.4) + f(.8)(.4) + \ldots + f(3.2)(.4) + f(3.6)(.4)$$

$\left(\Delta x = \frac{4-0}{10} = .4\right)$

8) Repeat steps 1 through 6 again letting the number of subintervals increase to n = 50. (Note: This takes some time to draw. So, while one machine is working on that picture, enter the function y = 9 - x ^ 2 in the other machine using 0 as the lower bound for x, 3 as the upper bound for x, -1 as the lower bound for y, and 9 as the upper bound for y. Select n = 10.)

9) When the picture of the first graph is completed for n = 50, discuss the refinement of the estimates from n = 4 to 10 to 50 asking students how good an estimate would result if n were increased to 100 or 10000 on to infinity. (Note: This package does not work for large values for n since the rectangles cannot be drawn. It is suggested that n = 50 is the largest value used for n.)

10) Introduce the limit notation for the left-edge rectangles, $\sum_{i=0}^{n-1} f(x_i)\Delta x$ and discuss.
(At this point you might want to draw the same functions on the board while students sketch them on graph paper at their seats. Discuss the use of the right edge of each rectangle for approximation purposes. The limit notation is a bit easier using the right edge method since the summation indices go from i=1 to n rather than from i=0 to n-1.) $\sum_{i=1}^{n} f(x_i)\Delta x$

11) Now refer back to both functions, $y = x^3$ and $y = 9 - x^2$. Discuss the increasing/decreasing nature of the functions and their concavity.

12) Ask the class to determine the conditions under which the approximating left-edge rectangles will be circumscribed and when they will be inscribed. Have the class answer the same questions for right-edge rectangles.

LEFT EDGE: IF f IS INCREASING (CONCAVE UP OR CONCAVE DOWN), THE RECTANGLES ARE INSCRIBED. IF f IS DECREASING (CONCAVE UP OR CONCAVE DOWN), THE RECTANGLES ARE CIRCUMSCRIBED.

RIGHT EDGE: IF f IS INCREASING, THE RECTANGLES ARE CIRCUMSCRIBED; IF f IS DECREASING, THE RECTANGLES ARE INSCRIBED.

13) Enter the function f(x) = sin(x) into the second machine, with 0 for the lower bound of x, 3.14 for the upper bound of x, -1 for the lower bound of y, and 1 for the upper bound of y. Let n = 10.

14) Have students use this picture to discuss their response to question 12 for left-edge rectangles. (You could also have students draw a quick sketch of the function f(x) = sin(x) and draw some approximating rectangles using the right edge of the rectangle . Then, have them discuss their response to question 12 for these rectangles.)

15) Enter the linear function f(x) = -.5*x+3 with -1 the lower limit for x, 6 the upper limit for x, 0 the lower limit for y, 3 the upper limit for y, and n = 10.

16) Ask students if they can come up with an easier way of finding the area under the curve. (Either using the area of a trapezoid or by composing the area of a rectangle and the area of a triangle.)

17) (Optional)
Use the Continue Option (C) for y = x ^ 2 which should still be on the first machine. Then enter the Quit Option (Q). Enter the Same Function Different Limits Option (S). Using the original boundaries of 0 to 4 for x and -1 to 16 for y, select the Midpoint Option (M) to show students how approximations for the area under the curve are calculated using the function value at the midpoint of the approximating rectangle.

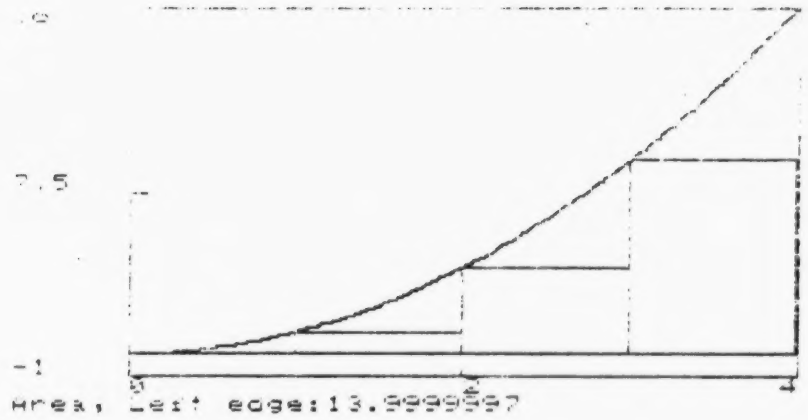
Enter the Continue Option(C) and select the Trapezoidal Option (T) to show students how approximations for the area under the curve are calculated using trapezoids instead of rectangles. Repeat the procedure to select the Simpson's Rule Option (S). This is the hardest option to see on the screen since arcs of parabolas are used for the approximation process.

Using the same function run the program again for n = 50. Use the Midpoint Method Option (M), the Trapezoidal Rule Option (T), or the Simpson's Rule Option (S). Discuss the refinement of the approximation from the left-edge rectangle to Simpson's Rule. Even if you cannot discuss the associated formulas for these methods at this time, students can at least appreciate the refinement that each of these provides over the rectangular approximation method.

18) Assign the homework assignment given with this lesson or a comparable lesson in your calculus text. Use the computer the next day to check the students' graphs and their rough estimates.

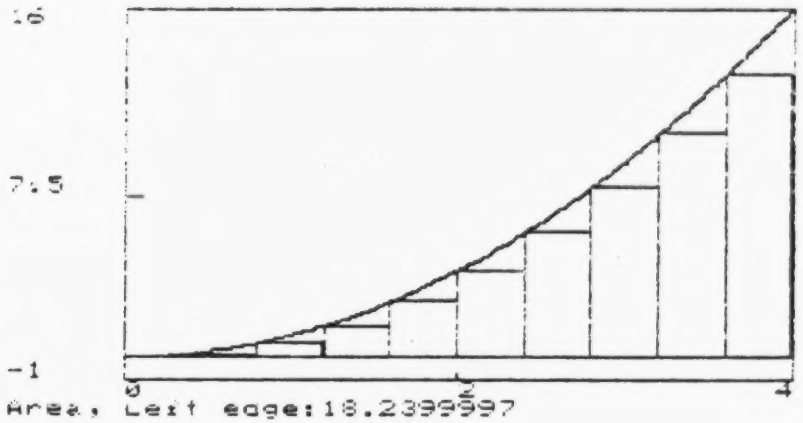
$f(x) = x^2$

$n = 4$



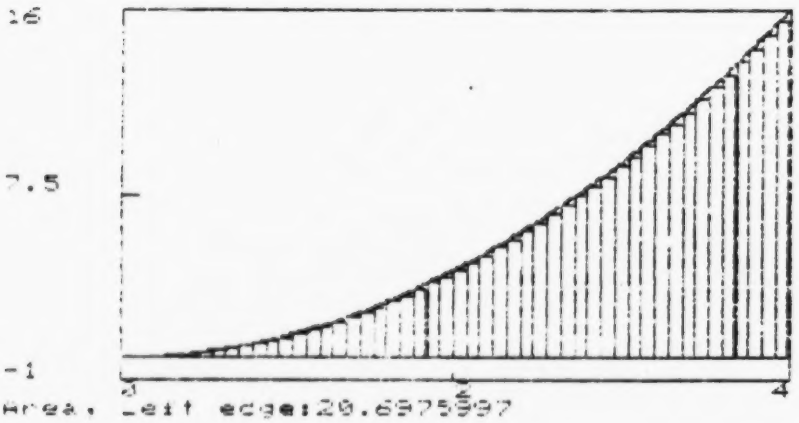
$f(x) = x^2$

$n = 10$

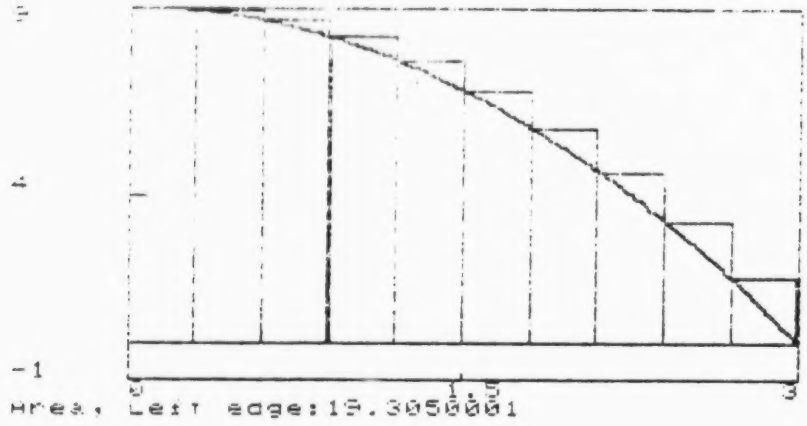


$f(x) = x^2$

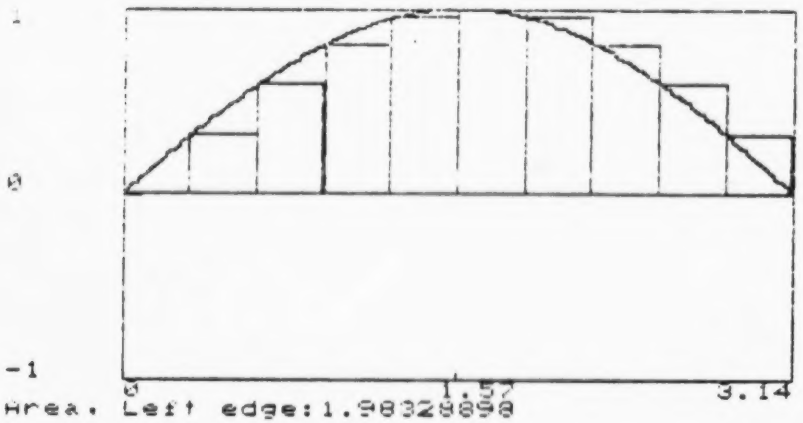
$n = 50$



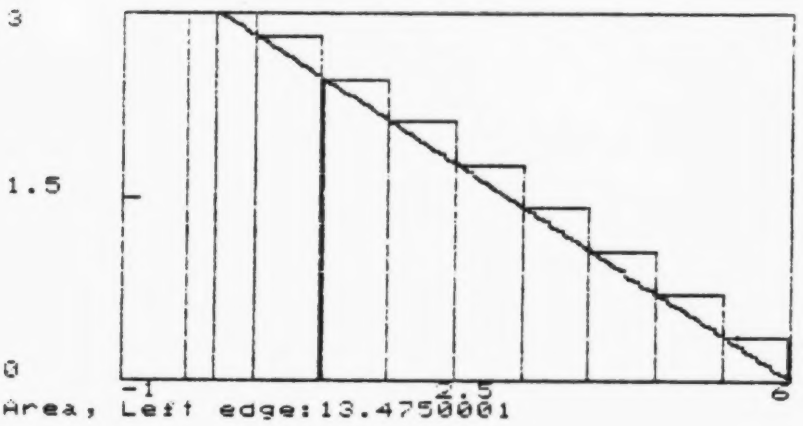
$f(x) = 9 - x^2$



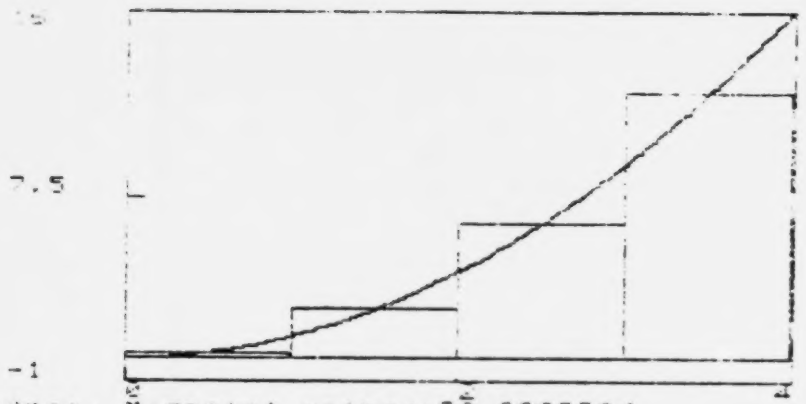
$f(x) = \sin(x)$



$f(x) = -.5x + 3$

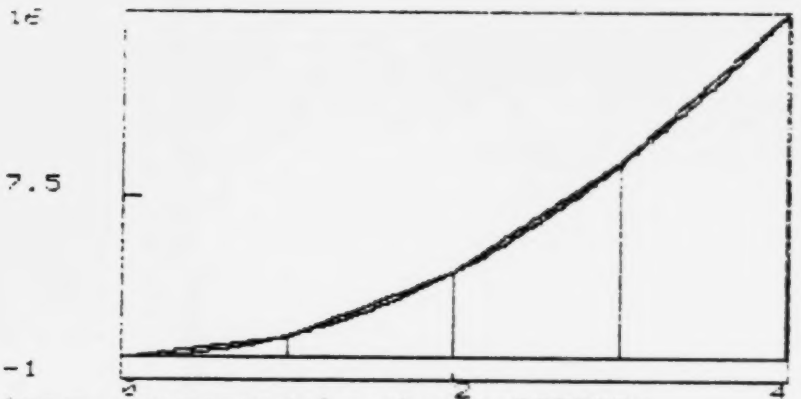


$f(x) = x^2$



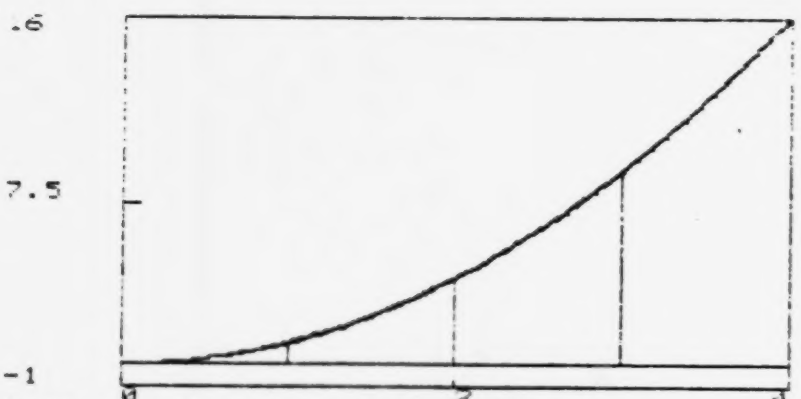
Area, Midpoint method: 20.9999996

$f(x) = x^2$



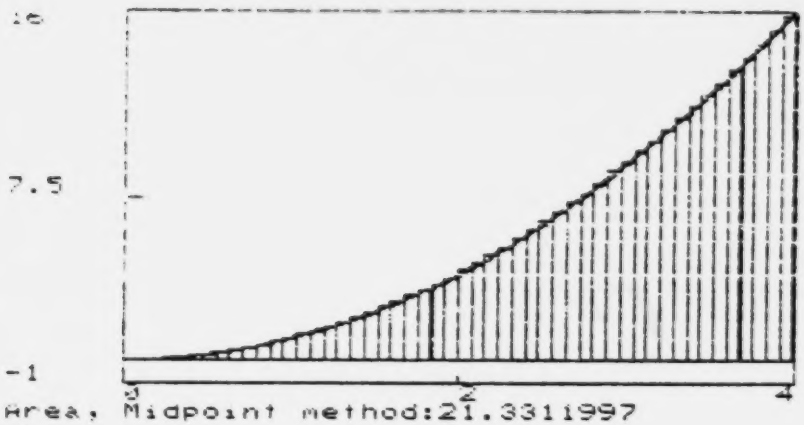
Area, Trapezoidal rule: 21.9999996

$f(x) = x^2$



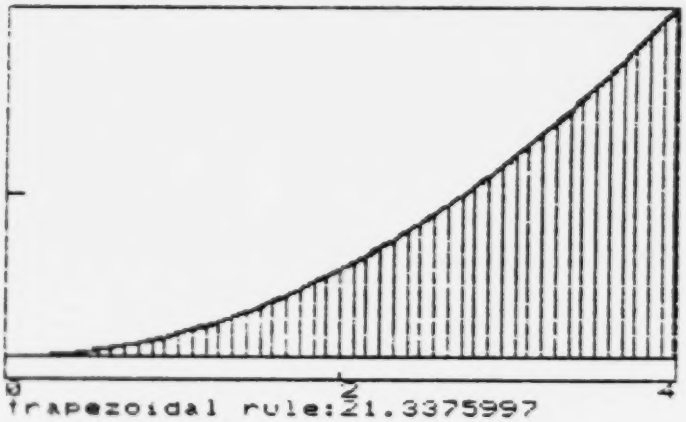
Area, Simpson's rule: 21.3333329

$f(x)=x^2$



Area, Midpoint method:21.3311997

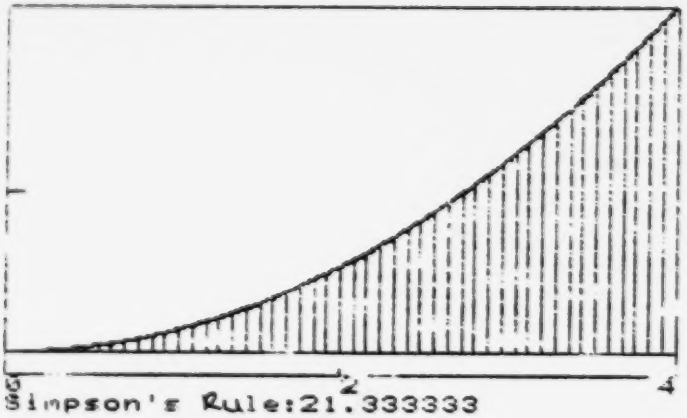
$f(x)=x^2$



Area, trapezoidal rule:21.3375997

USES TRAPEZOIDS

$f(x)=x^2$



Area, Simpson's Rule:21.333333

USES SECTIONS OF PARABOLAS

HOMEWORK EXERCISES

For each of the following functions,
- a) conatruct its graph over the given interval.
- b) draw the approximating rectanglea using either left-edge or right-edge rectanglea.
- c) write the estimation of the area under the curve using summation notation with n equal to 5.
- d) using your calculator, evaluate the result.

1) $f(x) = x + 4$ $[-1, 4]$

2) $f(x) = x^2 + 2$ $[0, 5]$

3) $f(x) = x^3 + 1$ $[1, 2]$

4) $f(x) = \cos(x)$ $[0, 1.57]$

5) $f(x) = \sqrt{1-x2}$ $[0, 1]$

6) $f(x) = 1/x$ $[1, 6]$

ANSWER KEY
HOMEWORK EXERCISES

For each of the following functions,
a) construct its graph over the given interval.
b) draw the approximating rectangles using either left-edge or right-edge rectangles.
c) write the estimation of the area under the curve using summation notation with n equal to 5.
d) using your calculator, evaluate the result.

1) $f(x) = x + 4$ $[-1, 4]$

2) $f(x) = x^2 + 2$ $[0, 5]$

3) $f(x) = x^3 + 1$ $[1, 2]$

4) $f(x) = \cos(x)$ $[0, 1.57]$

5) $f(x) = \sqrt{1-x2}$ $[0, 1]$

6) $f(x) = 1/x$ $[1, 6]$

1) $\sum_{i=0}^{4} f(x_i)\Delta x = \sum_{i=0}^{4} (x_i+4)\Delta x = 3+4+5+6+7 = 25$
$\Delta x = 4-(-1)/5 = 1$

2) $\sum_{i=0}^{4} f(x_i)\Delta x = \sum_{i=0}^{4} (x_i^2+2)\Delta x = 2+3+6+11+18 = 40$
$\Delta x = (5-0)/5 = 1$

3) $\sum_{i=0}^{4} f(x_i)\Delta x = \sum_{i=0}^{4} (x_i^3+1)\Delta x = .4 + .5456 + .7488 + 1.0192 + 1.3664 = 4.08$
$\Delta x = (2-1)/5 = .2$
(NOTE: $x_0 = 1$, $x_1 = 1.2$, $x_2 = 1.4$, $x_3 = 1.6$, $x_4 = 1.8$)

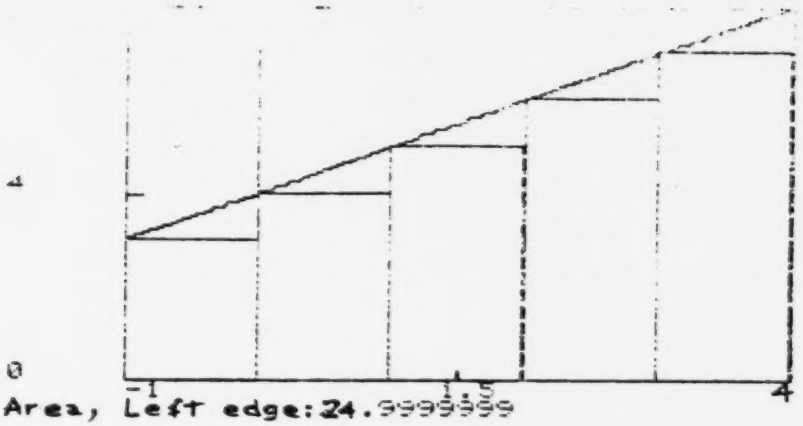
4) $\sum_{i=0}^{4} f(x_i)\Delta x = \sum_{i=0}^{4} \cos(x_i)\Delta x = .314 + .2986472 + .2540911 + .1846859 + .0972156 = 1.1486448$
$\Delta x = (1.57-0)/5 = .314$
(NOTE: $x_0 = 0$, $x_1 = .314$, $x_2 = .628$, $x_3 = .942$, $x_4 = 1.25$

5) $\sum_{i=0}^{4} f(x_i)\Delta x = \sum_{i=0}^{4} (\sqrt{1-x_i^2})\Delta x = .2 + .1959518 + .1833033 + .16 + .12 = .9592621$
$\Delta x = (1-0)/5 = .2$
(NOTE: $x_0 = 0$, $x_1 = .2$, $x_2 = .4$, $x_3 = .6$, $x_4 = .8$)

6) $\Delta x = (6-1)/5 = 1$ $x_0 = 1$, $x_1 = 2$, $x_2 = 3$, $x_3 = 4$, $x_4 = 5$ $\sum_{i=0}^{4} \frac{1}{x_i}\Delta x = 2.283333$

1)

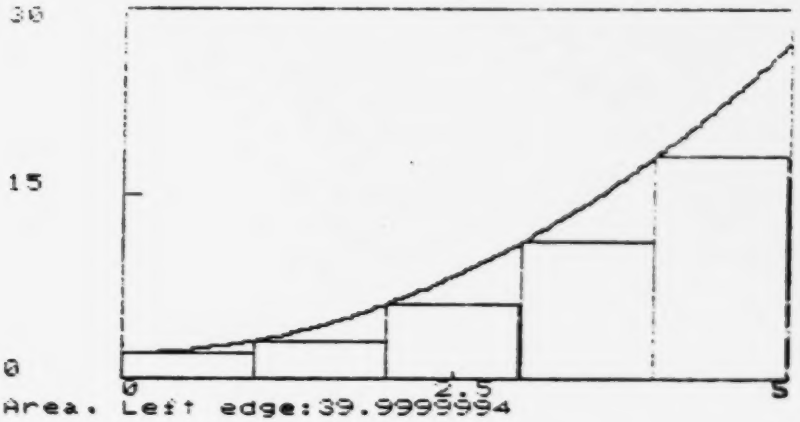
$f(x) = x + 4$



Area, Left edge: 24.9999999

2)

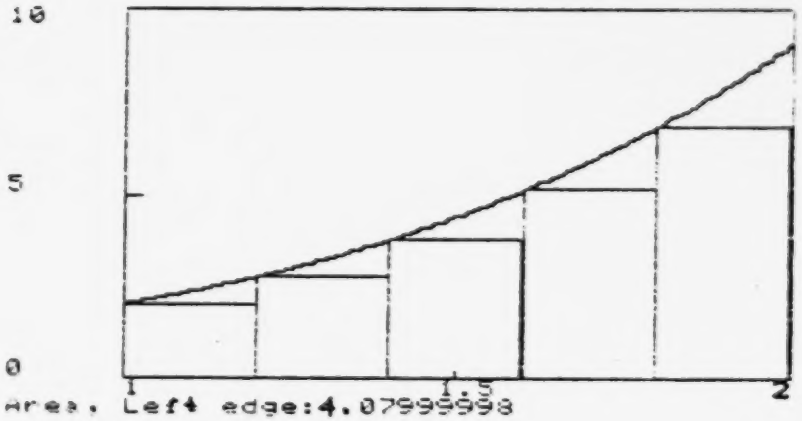
$f(x) = x^2 + 2$



Area, Left edge: 39.9999994

3)

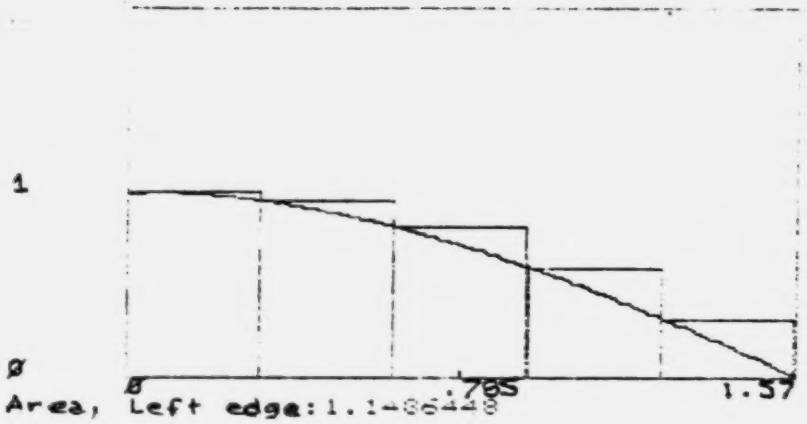
$f(x) = x^3 + 1$



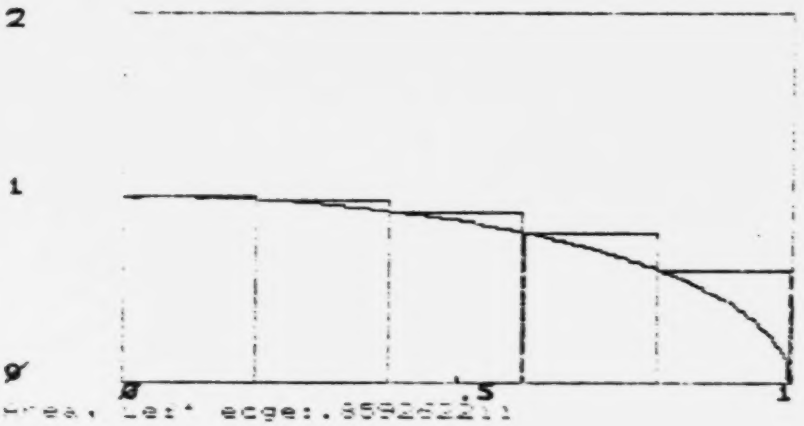
Area, Left edge: 4.07999998

4)

$f(x) = \cos(x)$



5) $f(x) = \sqrt{1-x^2}$



6)

$f(x) = \frac{1}{x}$